

Quantum Computing in Hybrid Systems and Algorithmic Complexity

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Massachusetts Institute of Technology, Cambridge, USA

Abstract—SOWKRXJK LW LV IXOO\ LPSRVVLEOH WR

HQVXUH WKDW D VRIWZDUH V\ VWHPV LV TXLWH VHFUXH GHYHORSLOJ DQ DFFHSWDEOH VHFUXH VRIWZDUHV\ VWHPV DQDFRQYHQLHQWSODWIRUPLVQRW XQUHDFKDEOH, QWKL VSDSHUZHDWWHPSWWRDQQDO\ JHVRIWZDUHGHYHORSPH QWOLIHF\ FOH6'& PRGHOV IURP WKH KDUGZDUH V\ VWHPV DQG FLUFXLWV SRLQW RI YLHZ 7R GDWHWKH6'& PRGHOVSD\ PHUHO\ DWWHQWLRQRWRWK HVRIWZDUHVHFUXULW\ IURPWKHVRIWZDUHSHUVSHFWLYHV, QWKL VSDSHUZHS UHVHQQWZHIHDWXUHV IRU 6'& VWDJHV WR HPSKDV\ JH WKH UROH RI V\ VWHPV DQG FLUFXLWV LQ GHYHORSLOJ VHFUXH VRIWZDUHV\ VWHPWKURXJKWKH VRIWZDUHGHYHORSPHQW VWDJHVWKHSRLQW WKDWKDVQRWEHHQFRQVLGHUHG SUHYLRXVO\ LQWKH6'& PRGHOV

Keywords—6\ VWHPV DQG FLUFXLWV VHFUXULW\ VRIWZDUH VHFUXULW\ VRIWZDUHSURFHVVHQJLQHHULQJ6'&66'&

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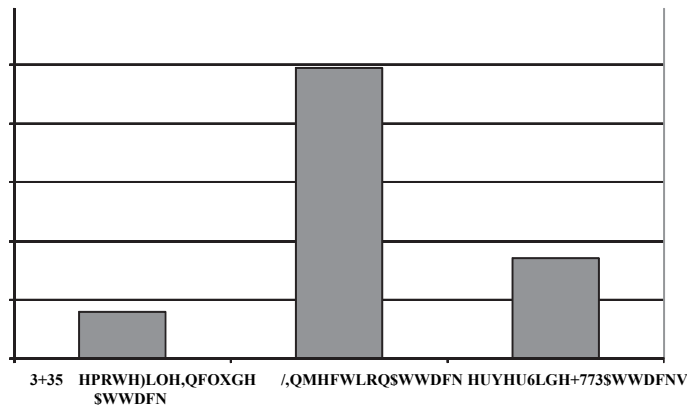
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