Adaptive Hybrid Approaches for Complex System Modeling

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***Abstract*²** 7KLVSDSHUSURVHQWVQXPHULFDOPRGHOEDVHGRQILQLWH GLVFUHWHHOHPHQWPHWKRGIRUDQDO\VLVRIWKHVW UXFWXUDOUHVSRQVHRIGU\ VWRQH PDVRQU\ VWUXFWXUHV XQGHU VWDWLF DQG G\QDPLF ORDGV 0RUH SUHFLVHO\HDFKGLVFUHWHVWRQHEORFNLVGLVFUHWL]HGE\ILQLWHHOHPHQWV 0DWHULDO QRQOLQHDULW\ LQFOXGLQJ IUDFWXUH DQG IUDJPHQWDWLRQ RI GLVFUHWHHOHPHQWVDVZHOODVF\FOLFEHKDYLRUGXULQJ\QDPLFORDGDUH FRQVLGHUHGWKURXJKFRQWDFWHOHPHQWVZKLFKDUHLPSOHPHQWHGZLWKLQD ILQLWHHOHPHQWPHVK7KHDSSOLFDWLRQRIWKHPRGHOZDVFRQGXFWHGRQ VHYHUDOH[DPSOHVRIWKHVWUXFWXUHV7KHSHUIRUPHGDQDO\VLV KLJK DFFXUDF\ RI WKH QXPHULFDO UHVXOWV LQ FRPSDULVRQ ZLWK WKH H[SHULPHQWDO RQHV DQG GHPRQVWUDWHV WKH SRWHQWLDO RI WKH ILQLWH GLVFUHWHHOHPHQWPHWKRGIRUPRGHOOLQJRIWKHUH VSRQVHRIGU\\VWRQH PDVRQU\\VWUXFWXUHV

***Keywords*²**)LQLWHGLVFUHWH HOHPHQW PHWKRG GU\\ VWRQH PDVRQU\\ VWUXFWXUHVVWDWLFORDGG\QDPLFORDG

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$ ODUJHSDUWRIFXOWXUDOKHULWDJHDOORYHUWKH ZRUOGDUH LQFUHDVLQJ WKHLU UHVLVWDQFH PDQ\ RI GU\ VWRQH KLVWRULFDO VWUXFWXUHVZHUHIXUWKHUVWUHQJWKHQHGE\V WHHOFODPSVDQGEROWV ,QRUGHUWRHYDOXDWHWKHUHVLVWDQFHRIWKH VHVWUXFWXUHVDQGWR EHDEOHWRSUHVHUYHWKHFXOWXUDOKHULWDJHL WLVQHFHVVDU\WR GHYHORSDQXPHULFDOPRGHOZKLFKFRXOGWDNHLQ WRDFFRXQWDOO WKHHIIHFWVRFFXUULQJLQGU\VWRQHPDVRQU\VWU XFWXUHVLQFOXGLQJ WKHIUDJPHQWDWLRQRIWKEORFNVDQGQROQH DUEHKDYLRURI VWHHOFODPSVDQGEROWVGXULQJ\QDPLFORDGLQ J

ä1LNROLüDQG1äLYDOMLüDUHZLWKWKH8QLYHUVLW\RI6OLW) DFXOW\RI&LYLO (QJLQHHULQJ $UFKLWHFWXUH DQG *HRGHV\ 6OLW +5&URDWLD SKRQH ID[HPDLO]HOMDQDQLNROLF#JUDGVWKU QLNROQD]LYDOMLF#JUDGVWKU

+ 6PROMDQRYLü LV ZLWK WKH 8QLYHUVLW\ RI 6OLW)DFXOW\ RI &LYLO (QJLQHHULQJ $UFKLWHFWXUH DQG *HRGHV\ 6OLW +5&URDWLD SKRQH ID[HPDLO KUYRMHVPROMDQRYLF#JUDGVWKU

DSSURDFKLVVXLWDEOHIRUPRGHOOLQJGLIIHUHQW W\SHVRIQRQOLQHDU EHKDYLRU LQFOXGLQJ ODUJH GLVSODFHPHQW DQG URWDWLRQ ZLWK FRPSOHWHGHWDFKPHQWRIEORFNV

,,7<3(62)67((/&/$036$1'%2/76

+LVWRULFDO GU\ VWRQH PDVRQU\ VWUXFWXUHV DUH FRPPRQ\ VWUHQJWKHQHG ZLWK VWHHO FODPSV HPEHGGHG RQ WKH ODWHUDO VXUIDFHRIWKHVWUXFWXUH)LJDRURQWKHWRSVLG HRIVWRQH EORFNV)LJEDVZHOODVZLWKVWHHOEROWV)LJF

D

E

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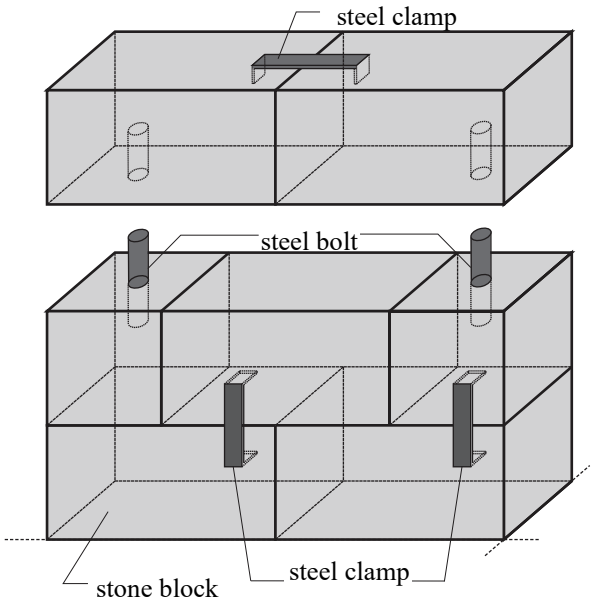
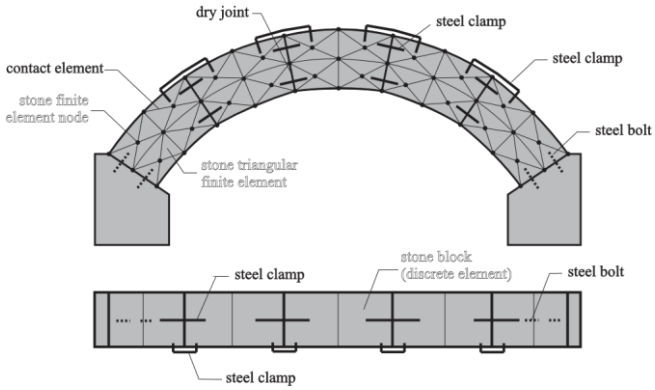
)LJ6WHHOFODPSVDQGEROWVDVWHHOFODPSHPEHGGHGRQWKHODWHUDO

&ODPSVDQGEROWVDUHHPEHGGHGLQWRWKHSUHYLRXVO\PDGH KROHVLQVWRQHEORFNVWKDWDUHVXEVHTXHQWO\EDFNILOOHGZLWK VRPHLQILOOPDWHULDOOLNHSOXPE &ODPSVDUHPRVWFRPPRQO\XVHGLQVWUHQJWKHQLQJRIGU\ VWRQHZDOOVDQGGU\VWRQHDUFKHVE\WHQVLRQE HDULQJFDSDFLW\ ZKLOHVWHHOEROWVDUHXVHGIRUFRQHFWLQJWKHFDSLWDOVDQG FROXPQVRUFDSLWDOVDQGXSSHUEHDPV)LJFDQGWH\ GRPLQDQWO\KDYHVKHDUEHDULQJFDSDFLW\ 'XHWRWKHSUHVHQFHRIPDSDUDPHWHUVZKLFKHI IHFWRQ EHKDYLRURIFODPSVDQGEROWVLQGU\VWRQHPDVR QU\VWUXFWXUHV VXFKDVWKHHODVWLFSURSHUWLHVRIVWRQHDQG VWHHOWKHZLGWKDQG GHSWK RI WKH KROH

LQWR ZKLFK WKH FODPSV DQG EROWV DUH HPEHGGHGHODVWLFSURSHUWLHVRILQILOOPDWHULDOJHRPHWU\RI FODPSVDQGEROWVHWFLWVYHU\GLIILFXOWWRGH YHORSQXPHULFDO PRGHO ZKLFK FDQ WDNH LQWR DFFRXQW DOO W\SHV RI IDLOXUH PHFKDQLVPVDQGHVSHFLDOO\WKHLQIOXHQFHRIOR FDOLQWHUDFWLRQ EHWZHHQWKHEROWRUFODPSRQRQHVLGHDQGPDVR QU\EORFNRQ 6FKHPDWLFSUHVHQWDWLRQVRIVWHHOFODPSVDQG VWHHOEROWV PRGHOOHGLQWKLVSDSHUHVKRZQLQ)LJ



) LJ6WHHOFODPSVDQGEROWV



)LJ'LVFUHWL]DWLRQRIGU\VWRQHVWUXFWXUHZLWK VWHHOFODPSVDQG EROWV

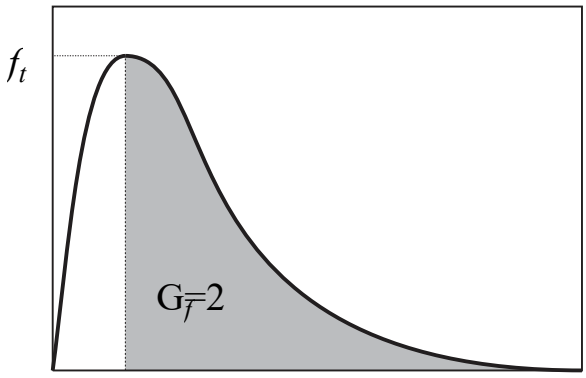
,,, ',6&5(7,=$7,212)$'5<6721(6758&785(:,7+ 67((/&/$036$1'%2/76

OLQHDULW\IUDFWXUHDQGIUDJPHQWDWLRQDUHFR QVLGHUHGWKURXJK WKH FRQWDFW HOHPHQWV ZKLFK DUH SODFHG ZLWKLQ WKH ILQLWH HOHPHQWPHVKRIHDFKEORFN

7KHVWHHOFODPSVDQGVWHHOEROWVZHUHPRGHO OHGZLWKRQH GLPHQVLRQDO HOHPHQWV ZKLFK FDQ EH SODFHG LQ DUELWUDU\ SRVLWLRQVLQVLGHWKHVWRQHILQLWHHOHPHQWV 'LVFUHWL]DWLRQ RI GU\ VWRQH PDVRQU\ VWUXFWXUH ZLWK HPEHGGHGVWHHOFODPSVDQGEROWVLVVKRZQLQ)LJ

,9 121/,1($50$7(5,$/02'(/)6721(%/2&.6

DUHDXQGHUWKHVWUHVVVWUDLQFXUYHFRQVLVWVRIWZRSDUWV)LJ SDUWIRUPRGHOOLQJRIWKHVWRQHEHKDYLRUXSWRWKHFUDFN RSHQLQJDQGSDUWZKLFKUHSUHVHQWVVWUDLQVRIWHQLQJIWHUWKH WHQVLOHVWUHQJWKLVH[FHHGG7KHDVVXPSWLRQRIWKHGLVFUHWH FUDFNPRGHOVWKDWWKHFUDFNVFRLQFLGHZLWKWKHILQLWHHOHPHQW VHSDUDWLRQVL]Hį



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)LJ6WUDLQVRIWHQLQJVWUHVVVWUDLQDQGVWUHVVGLVSODFHPHQWFXUYHV

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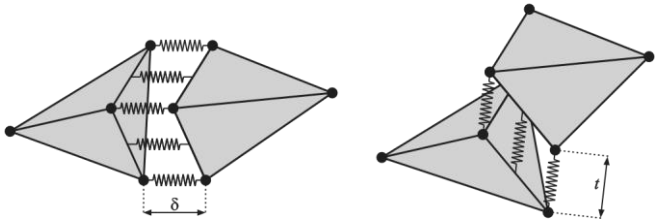
A.Steel material model

LVVKRZQLQ)LJE



)LJ1RUPDODQGVKHDUVSULQJVEHWZHHQWKHILQLWHHOHPHQWV

ORDGLQJ.DWR

H[SUHVVLRQV
GXULQJXQORDGLQJ)LJEFXUYH

ZKHUHE_sLV<RXQJ VPRGXOXVRIVWHHO
GXULQJQHJDWLYHORDGLQJ)LJEFXUYH

V$_{sc}$ f_y «»

«¬E_B f_y H$_{sc}$ H$_{sh}$ H$_y$ a »¼

ZKHUHE_B E_s ORJ H$_{sh}$ H$_y$ a E_s E_s E_B
GXULQJUHORDGLQJXQORDGLQJ)LJEFXUYH

ZKHUHι_{pm}LVWKHPLQLPXPYDOXHRIι_{sc}LQLWVORDGLQJ
KLVWRU\
GXULQJUHORDGLQJ)LJEFXUYH

V$_{sc}$ f_y V$_{pm}$ f_y ª« a ^aa ` »º

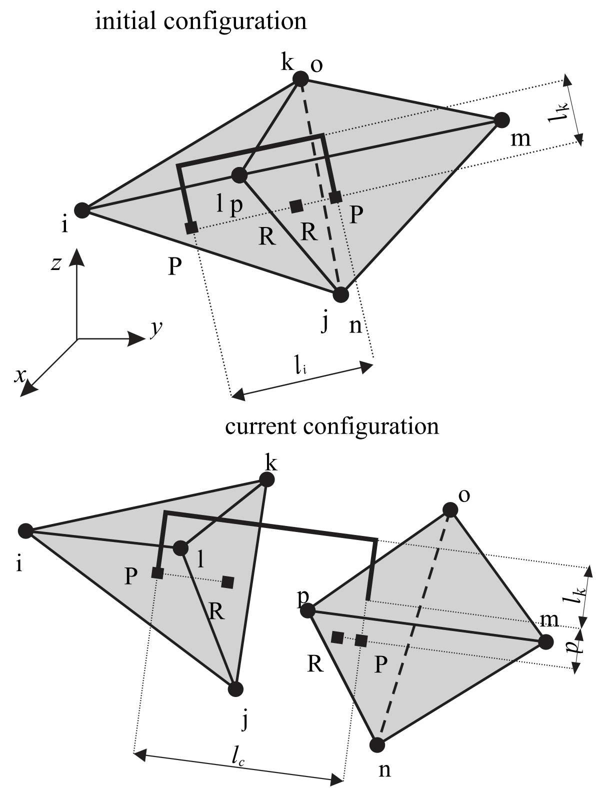
«¬E_B f_y H$_{sc}$ H$_{sh}$ H$_y$ a »¼

B.Steel Clamps

7KHVWHHOFODPSZDVGHILQHGE\LWVILUVWSRLQW3WKHHQG
SRLQW3DQGWKHDQFKRUDJHOHQJWKl_k)LJ7KHLQWHUVHFWLRQ

EHWZHHQWKHEORFNVHGJHVDQGOLQHVHJPHQW33
JLYHVWKH

UHIHUHQWVSRLQWV5DQG57KHVWUDLQRIDVWHHO
FODPSLQ
DUELWUDU\WLPHVWHSFDQEHREWDLQHGIURPWKHF
RRUGLQDWHVRI
WKHVHSRLQWVLQFXUUHQWFRQILJXUDWLRQ



)LJ6WHHOFODPSLQLQLWLDODQGFXUUHQWFRQILJXUDWLRQ

&RRUGLQDWHV RI SRLQW 3 LQ FXUUHQW
FRQILJXUDWLRQ DUH
REWDLQHGDFFRUGLQJWR

x_c Ax_{ic} Bx_{jc} Cx_{kc} Dx_{lc}

y_c Ay_{ic} By_{jc} Cy_{kc} Dy_{lc} z_c Az_{ic} Bz_{jc} Cz_{kc} Dz_{lc}

ZKHUH$x_c$$y_cDQGz_c$DUHFRRUGLQDWHVRISDUWLFXODU
QRGHLQFXUUHQW
FRQILJXUDWLRQ8QNQRZQFRHIILFLHQWV A B CDQG
DFDQEH
REWDLQHGIURP

A $\frac{V_i}{V}$ B $\frac{V_j}{V}$ C $\frac{V_k}{V}$ D $\frac{V_l}{V}$

ZKHUH*V*LVWKHYROXPHRIWKHFRUUHVSRQGLQJWH WUDKHGURQZKLOH
$V_iV_jV_k$DQGV_lDUHYROXPHRIWHWUDKHGURQVGHILQH GE\SRLQWV
*jkliklijl*DQG*ijk*UHVSHFWLYHO\&RRUGLQDWHVRISRLQ W3
5DQG5LQFXUUHQWFRQILJXUDWLRQFDQEHREWDLQ HGLQVLPLODU
ZD\
6WUDLQRIVWHHOFODPSLVJLYHQE\

$$H_{sc} = \frac{l_c - l_i}{l_i}$$

ZKHUH l_cDQG
l_iDUHOHQJWKVRIDVWHHOFODPSLQFXUUHQWDQG LQLWLDOFRQILJXUDWLRQVUHVSHFWLYHO\\)URPREWDLQHGVWUDLQLWLV
SRVVLEOHWRFDOFXODWHVWUHVV$_{sc}$RIWKHVWHHO FODPSIURPWKHVWHHO PDWHULDOPRGHO
7KHLQIOXHQFHRIWDQJHQWLDOVHSDUDWLRQ *p*LVDSSUR[LPDWHO\
WDNHQLQWRDFFRXQWWKURXJKDUHGXFWLRQRIV WUHVV $_{sc}$**c** JLYHQE\\

$$V_{sc}\mathbf{c} = zV_{sc}$$

ZKHUH*z*LVWKHVFDOLQJIXQFWLRQ7KHVFDOLQJIXQF WLRQLVHTXDO
WRRQHZKHQWKHUHLVQRVKHDUVHSDUDWLRQZKL OHLWLVHTXDOWR
]HURZKHQWKHVKHDUVHSDUDWLRQLVHTXDOWR *l_k*)RUVKHDU
VHSDUDWLRQ*pl_k*WKHVFDOLQJIXQFWLRQGHSHQGVR QWKHHODVWLF
SURSHUWLHVRIVWRQHDQGDVWHHOFODPSWKHZLG WKRIWKHKROHLQ
ZKLFKWKHVWHHOFODPSLVHPEHGGHGWKHHODVWL FSURSHUWLHVRI
WKHLQILOOPDWHULDOWKHFURVVVHFWLRQDUHDR IWKHFODPSHWF,Q
D

$$z = \frac{e\ D}{e\ D}$$

D

ZKHUHYDULDEOH*D=D(p)*LVGHWHUPLQHGDFFRUGLQ JWR

$$Dp \to \begin{cases} - & \text{IRU} \quad p \\ \circ & \text{IRU} \quad p \text{ t } l_k \end{cases}$$

°–pl_k RWKHU

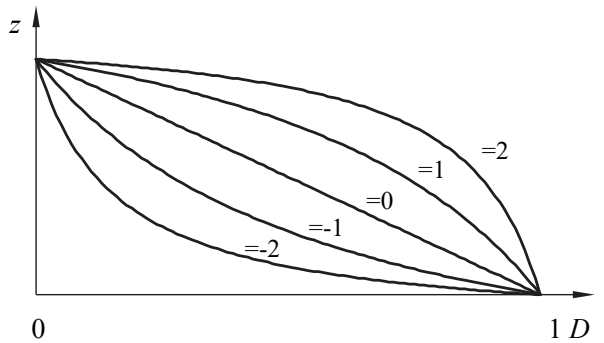
7KH VKDSHRI IXQFWLRQ *z* IRUGLIIHUHQW YDOXHV RI VKDSH SDUDPHWHU*ĮLVVKRZQ)LJ
,QDEVHQFHRIH[SHULPHQWDOUHVXOWVWKHVKDSH SDUDPHWHU*Į*=
Į$_c$IRUVWHHOFODPSVFDQEHLQLWLDOO\\VHWWR]HUR ZKLFKOHDGVWRD
OLQHDUO\GHFUHDVLQJIXQFWLRQ,IH[SHULPHQWDO UHVXOWVH[LVW
SDUDPHWHU*Į*$_c$FDQEHFKRVHQWREHVWILWH[SHULPH QWDOGDWD
7KHIRUFHLQDVWHHOFODPS)LJDLVJLYHQE\\

$$\mathbf{I}_{sc} = \mathbf{I}_{sc}\, A_{sc} V_{sc}\mathbf{c}$$

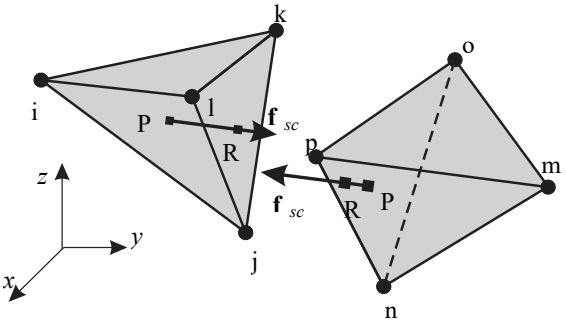
ZKHUH A_{sc}LVWKHFURVVVHFWLRQDODUHDRIFODPS)RUFHV$\mathbf{I}_{sc}$DQG$\mathbf{I}_{sc}$DFWLQJLQSRLQWV3DQG3DUHGLV WULEXWHG
LQWRWKHQRGHVRIWKHSDUHQWVWRQHWULDQJXO DUILQLWHHOHPHQWLQ
WKHIRUPRIHTXLYDOHQWQRGDOIRUFHV)LJE

A.Behavior of Clamps under Monotonic Loading

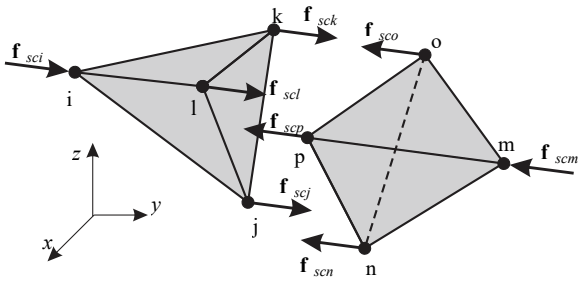
7KHEHKDYLRURIFODPSVXQGHUPRQRWRQLFORDG LQJZDV
SHUIRUPHGQWZRULJLGWHWUDKHGURQVFRQQHF WHGZLWKDVWHHO
FODPS)LJ7KHPDWHULDOFKDUDFWHULVWLFVRIWKH VWHHOFODPS DUHVKRZQLQ7DEOH,
7KHPRQRWRQLFDOO\LQFUHDVLQJORDGZDVSHUIR UPHGLQWHUV
RIFRQVWDQWYHORFLW\\$_xDQGv_y$LQSRLQW%9HORFLW \\$v_x$ZDVHTXDO v_x
PVZKLOHYHORFLW\\$v_y$ZDVYDULHGZLWKYDOXHVRI PVPVPVDQGPV



)LJ5HGXFWLRQIDFWRUIRUGLIIHUHQWYDOXHVRIVKDS HSDUDPHWHU*Į



(a)



(b)

)LJ6WHHOFODPSDIRUFHLQVWHHOFODPSEHTXLYDOHQWQRGDO

IRUFHV

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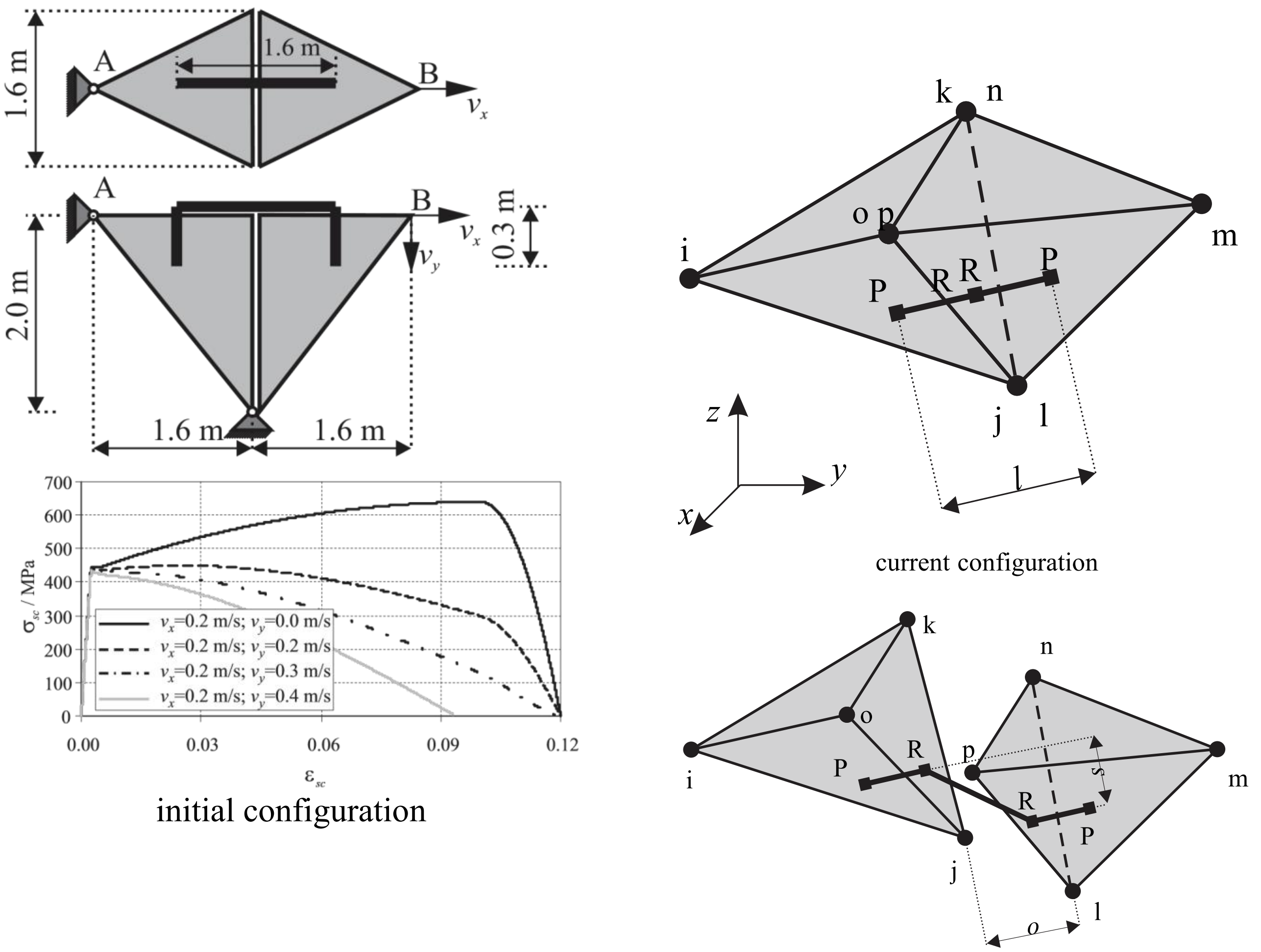
0$7(5,$/&+5&7(5,67,&62)67((/

6\PERO	4XDQWLW\	
E_s	0RGXOXVRIHODVWLFLW\	03D
fy	<LHOGVWUHVV	03D
fu	8OWLPDWHVWUHVV	03D
As	&URVVVHFWLRQDUHD	P
$\dot{I}_{sh}$	6WUDLQHQGRI\LHOG)	
$\dot{I}_u$	8OWLPDWHVWUDLQ	
$\dot{I}_{br}$	%UHDNVWUDLQ	

7KHVWUHVVVWUDLQUHODWLRQLQDVWHHOFODPSIRUDOOFDVHVRI ORDGLQJLVVKRZQLQ)LJ,WFDQEHVHHQWKDWIRUYHORFLWLHV v_yGLIIHUHQWIURP]HURWKHUHGXFWLRQLQVWUHVVRFFXUVGXHWRWKH H[WUDFWLQJRIWKHVWHHOFODPSIURPWKHVWRQHEORFN([WUDFWLQJ RIWKHVWHHOFODPSIURPWKHEORFNLQFUHDVHVZLWKWKHLQFUHDVH RIWKHYHORFLW\$v_y$WKLVOHDGVWRWKHDGGLWLRQDOUHGXFWLRQRIWKH VWUHVVLQWKHVWHHOFODPS



)LJ6WUHVVVWUDLQUHODWLRQLQVWHHOFODPSVIRUGLIIHUHQWFDVHVRI ORDGLQJ

7KHVWHHOEROWZDVGHILQHGE\LWVILUVWSRLQW3DQGWKHHQG)LJ6WHHOEROWLQLQLWLDODQGFXUUHQWFRQILJX
SRLQW37KHLQWHUVHFWLRQEHWZHHQWKHEORFNVHGJHVDQGOLQH

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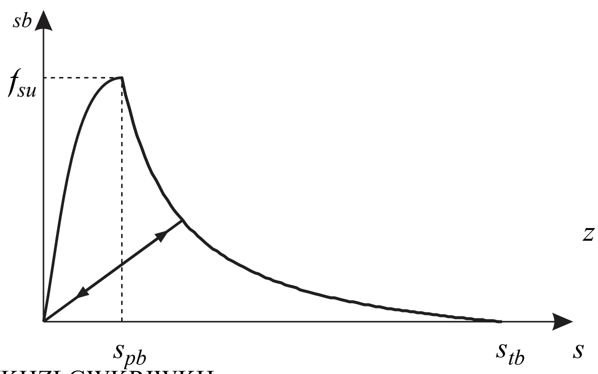
A.Geometry and Material Model of the Bolts

VHJPHQWJLYHVWKHUHIHUHQFHSRLQWV5DQG5)LJ7KH



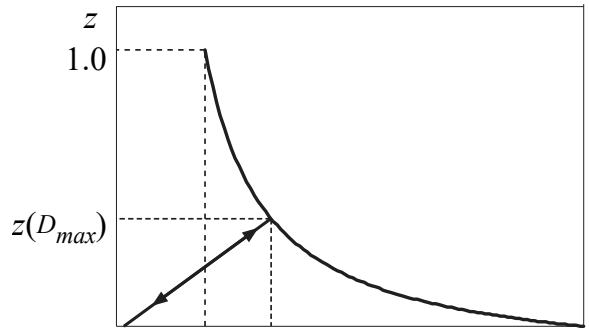
RQHODVWLFSURSHUWLHVRIVWRQHDQGDVWHHOEROWWKHZLGWKRIWKH

KROHLQZKLFKWKHVWHHOEROWLVHPEHGGHGWKHHODVWLFSURSHUWLHV
RIWKHLQILOOPDWHULDOWKHFURVVVHFWLRQDUHDRIWKHEROWHWF
DQGQHHGWREHGHWHUPLQHGH[SHULPHQWDOO\IRUHDFKSDUWLFXODU
FDVH

(a)

FRRUGLQDWHVRISRLQWV335DQG5LQFXUUHQWFRQILJXUDWLRQ
DUHREWDLQHGLQDVLPLODUZD\DVLWZDVVKRZQLQVWHHOFODPSV PRGHO
7DQJHQWLDOVHSDUDWLRQVLQGXFHVVKHDUVWUHVVHVIJ_{sb}LQWKHVWHHO EROW DQG DW
VHSDUDWLRQ s s_{pb} WKH VKHDU VWUHVV UHDFKHV LWV

PD[LPXP f_{su})LJ D :LWK LQFUHDVLQJ WDQJHQWLDO
VHSDUDWLRQ$s!s_{pb}$VKHDUVWUHVVGHFUHDVHVDQGDWVHSDUDWLRQ$s!s_{tb}$LW
GURSVWR]HURDQGWKHROWLVDVXPHGWREHEURNHQ
9DOXHVRI$f_{su}s_{pb}s_{tb}$DQGVKDSHRIIXQFWLRQVZKLFKGHILQH
WKHUHODWLRQEHWZHHQVKHDUVHSDUDWLRQDQGVKHDUVWUHVVGHSHQGV



s s_{pb} D_{max} s_{tb} (b)

)LJ0DWHULDOPRGHOLQWKHVWHHOEROWDVKHDUV
WUHVVHVYHUVXV
VKHDUVHSDUDWLRQEF\FOLFEHKDYLRU

7KHPD[LPXPYDOXHRIVKHDUVWUHVV
f_{su}LQVWHHOEROWLV
OLPLWHGZLWKVKHDUVWUHQJWKRIEROWPDWHULDO
Of_{sb}ZKLFKFDQEH
ZULWWHQDV

f_{su} d f_{sb}

DQGZLWKVKHDUVWUHVVLQEROWZKLFKFDXVHORF
DOFUXVKLQJRI
VWRQHDURXQGWKHROW7KHVKHDUVWUHQJWKRI
VWHHOf_{sb}FDQEH
GHWHUPLQHGIURPWKHWHQVLOHVWUHQJWKf_uE\DSS
O\LQJWKH9RQ
0LVHV\LHOGFULWHULRQZKLFKOHDGVWR

$f_{sb} f_u$ $\sqrt{/}$

ZULWWHQDV

f_{su} $\frac{dS}{\text{——}}$ d $d f_c$ /

ZKLFKOHDGVWR

f_{su} d f_c

ZKHUHdLVGLDPHWHURIEROW
,Q DFWXDO LPSOHPHQWDWLRQ IRU VHSDUDWLRQ
□_s_s_{pb} VKHDU

VWUHVVLVJLYHQE\

§ §

W$_{sb}$ ···· ss_{pb} ····© ss_{pb} · ,, ^{1} ·', ^{1} f_{su}

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ZKHUHYDOXHRIs_{pb}LVWUHDWHGDVLQSXWSDUDPHW
HU)RVHSDUDWLRQ
s_{pb}□_s_s_{tb}VKHDUVWUHVVLVDVVXPHGDV
W$_{sb}$ z f_{su}

ZKHUHzLVWKHVFDOLQJIXQFWLRQGHILQHGZLWKZK
HUH

- IRU s s_{pb}
°
D Ds ® IRU st_{tb}
°┼|s|s_{pb} s_{tb} s_{pb} RWKHU

7KHYDOXHRIWKHVKSHSDUDPHWHUI
I_{bs}QHHGVWREHFKRVHQ
WREHVWILWH[SHULPHQWDOGDWD,QWKHDEVHQFH
RIH[SHULPHQWDO
GDWDLWFDQEHVHWWRRLQLWLDOYDOXHI_{bs}
+RZHYHUFROODSVHRI
WKHVWRQHPDVRQU\VWUXFWXUHVXVXDOO\RFFXU
VGXHWRWKHORVVRI
WKHJOREDOVWDELOLW\DQGWKLVSDUDPHWHUKDV
QRLQIOXHQFHRQWKH
JOREDOVWUXFWXUDOEHKDYLRXU,IWKHFROODSVH
RIWKHVWUXFWXUH
FDXVHGE\WKHEUHDNLQJRIWKHHROWVSDUDPHWHU
I_{bs}KDVRQO\
LQIOXHQFHRQWKHVKDSHRIWKHORDGGLVSODFHPH
QWFXUYHLQ
VRIWHQLQJSKDVHEXWQRWWRWKHYDOXHRIFROOD
SVHORDG
7KH FRPSOHWH UHODWLRQVKLS IRU WKH VHDU
VWUHVV DV WKH
IXQFWLRQRIVKHDUVHSDUDWLRQFDQEHZULWWHQ
DV

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7KHLQIOXHQFHRIQRUPDOVHSDUDWLRQRLVDSSU R[LPDWHO\WDNHQ

LQWRDFFRXQWWKURXJKDUHGXFWLRQRIVWUHVV W $_{sb}$c JLYHQE\

$W_{sb}c\ zW_{sb}$

ZKHUH z LVWKHVFDOLQJIXQFWLRQ7KHVFDOLQJIXQF WLRQLVHTXDO WRRQHZKHQWKHUHLVQRQRUPDOVHSDUDWLRQZKL OHLWLVHTXDOWR]HURZKHQWKHQRUPDOVHSDUDWLRQLVHTXDOWR $l/2$ ZKHUH l LVWKH VWHHOEROWOHQJWK)RUQRUPDOVHSDUDWLRQ $ol/2$ WKHVFDOLQJ IXQFWLRQGHSHQGVRIHODVWLFSURSHUWLHVRIVWW RQHDQGDVWHHOEROW WKHZLGWKRIWKHKROHLQZKLFKWKHVWHHOEROW LVHPEHGGHGWKH HODVWLFSURSHUWLHVRIWKHLQILOOPDWHULDOW KHFURVVVHFWLRQDUHDRI WKHEROWHWF,QWKLVQXPHULFDOPRGHOWKHVFDO LQJIXQFWLRQLV DVVXPHGDFFRUGLQJWRZKHUHWKHYDULDEOH $D=Do$ LV GHWHUPLQHGDFFRUGLQJWR

$D\ Do\ o\ l$

&\FOLFEHKDYLRXURIEROW)LJLVDVVXPHGDV

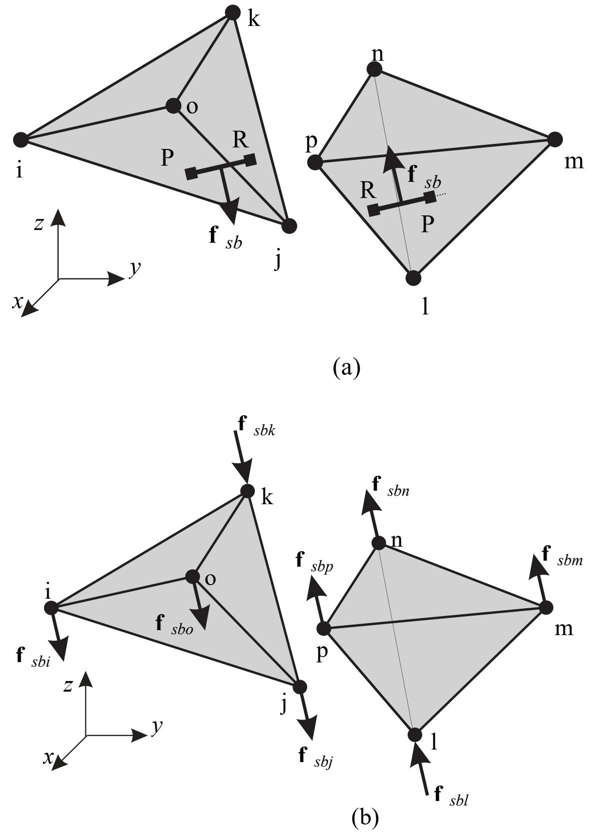
$$z\ z^{D}_{max} \quad \frac{Ds_{tb}\ \ s_{pb}\ \ s_{pb}}{D_{max}\, s_{tb}\ \ s_{pb}\ \ s_{pb}}$$

ZKHUH D_{max} LVWKHPD[LPXPYDOXHRI Ds LQLWVORDGLQJ KLVWRU\
6KHDUIRUFHLQWKHVWHHOEROW)LJDLVJLYHQE\

$\mathbf{I}_{sb}\ \mathbf{I}_{sb}\ A_{sb}W_{sb}$

ZKHUH A_{sb} LVFURVVVHFWLRQDUHDRIWKHEROW)RUFHV $\mathbf{I}_{sb}$ DQG $\mathbf{I}_{sb}$ ZKLFKDUHDVVXPHGWRDFWLQWKHFHQWUHRI

WKHEROWDQFKRUHGLQWKHVWRQHEORFNDUHGLV WULEXWHGLQWRWKH QRGHVRIWKHSDUHQWVWRQHWULDQJXODUILQLWH HOHPHQWLQWKHIRUP RIHTXLYDOHQWQRGDOIRUFHV)LJE



)LJ6WHHOEROWDIRUFHLQVWHHOEROWEHTXLYDOHQ WQRGDOIRUFHV

B.Behavior of Bolts under Monotonic Loading

7KHEHKDYLRXURIWKHPRGHORIEROWVXQGHUPR QRWRQLF ORDGLQJZDVSHUIRUPHGRQWZRULJLGWHWUDKHGU RQVFRQQHFWHG ZLWKVWHHOEROW)LJ7KHSDUDPHWHUVDSSOLHGLQ WKH QXPHULFDODQDO\VLVDUHVKRZQLQ7DEOH,,

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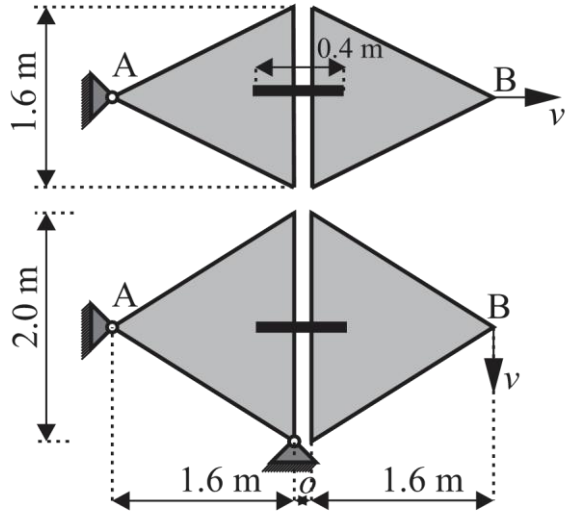
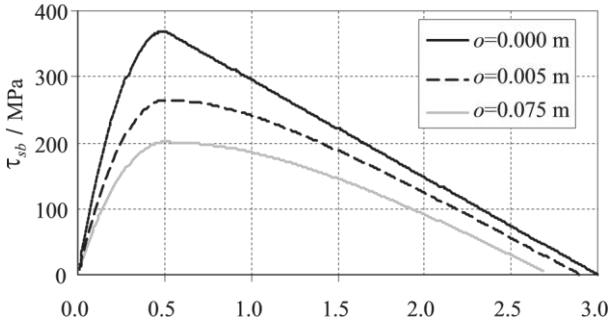
6\PERO	4XDQWLW\	
D	'LDPHWHU	PP
f_{tb}	7DQJHQWLDOVWUHQJWK	03D
s_{pb}	8OWLPDWHWDQJHQWLDOVHSDUDWLRQ	PP
s_{tb}	%UHDNWDQJHQWLDOVHSDUDWLRQ	PP

7KHPRQRWRQLFDOO\LQFUHDVLQJORDGZDVSHUIR UPHGLQWHUPV RIFRQVWDQWYHORFLW\ v PVLQSRLQW%,QLWLDOQRUPDO

GLVSODFHPHQW*o*LQQXPHULFDODQDO\VLVZDVYDULHGZLWKYDOXHV RIPPDQGP

6KHDUVWUHVVVKHDUVHSDUDWLRQHODWLRQVLQVWHHOEROWVIRU GLIIHUHQWLQLWLDOQRUPDOGLVSODFHPHQWDUHVKRZQLQ)LJ,W FDQEHVHHQWKDWLQFUHDVLQJWKHLQLWLDOQRUPDOGLVSODFHPHQWWKH UHGXFWLRQLQVKHDUVWUHVVHVDOVRLQFUHDVHV





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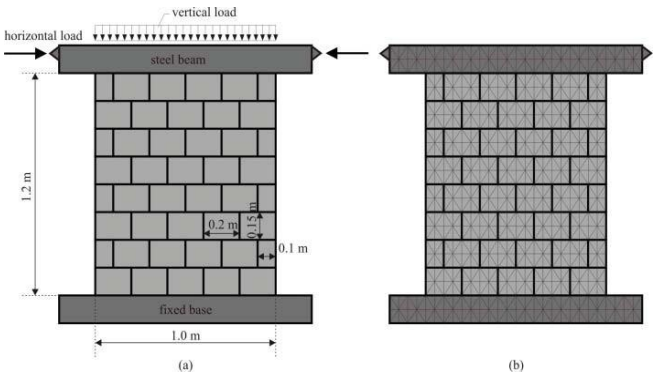
7KH ZDOO FRQVLVWHG RI WKH VDZQ VWRQH XQLWV ZLWK WKH GLPHQVLRQVRIPPOHQJWKîPPKHLJKWîPP ZLGWK&RQWDFWHOHPHQWVZKLFKSUHVHQWSRWHQWLDOFUDFNVLQ VWRQHXQLWVDUHLPSOHPHQWHGEHWZHHQWKHIL QLWHHOHPHQWPHVK 0HFKDQLFDO FKDUDFWHULVWLFV RI WKH FRQWDFW HOHPHQWV DUH <RXQJ¶V0RGXOXV*E 03DWHQVLOHVWUHQJWKft 03D FRPSUHVVLYHVWUHQJWKfc 03DIUDFWXUHHQHUJ\ GI$f

1PDQGVOLGLQJIULFWLRQ$μ 7KHF\FOLFORDGLQJKLVWRU\LV VKRZQLQ)LJ



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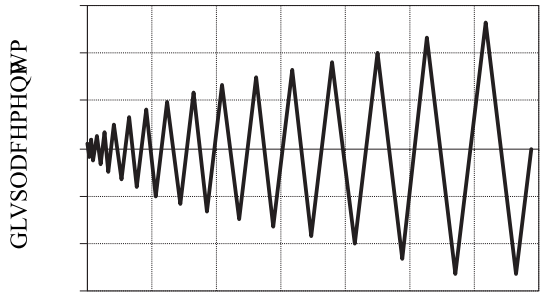

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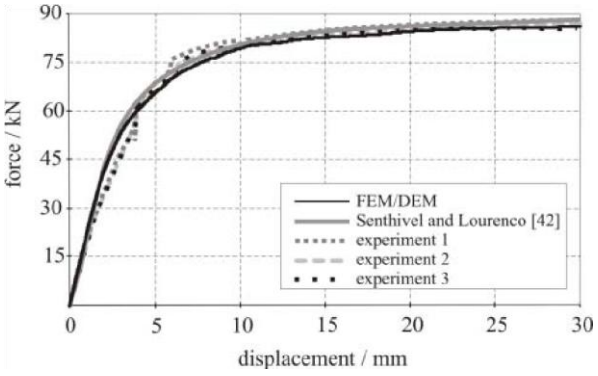

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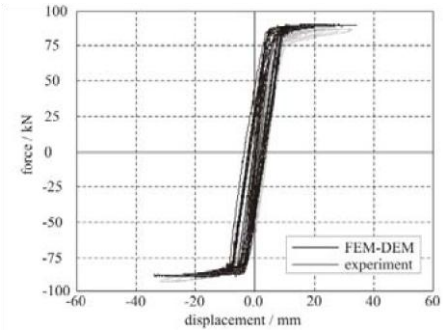

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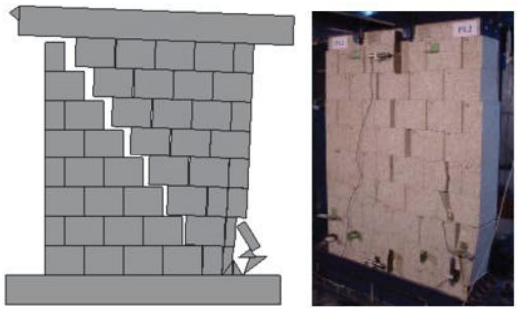

)LJ)DLOXUHSDWWHUQRIGU\PDVRQU\ZDOOXQHULFDO DQG H[SHULPHQWDO

9,,,&21&/86,21

7KLV SDSHU SUHVHQWV QXPHULFDO PRGHO IRU DQDO\VLV DQG SUHGLFWKHFRRODSVHRIGU\VWRQHPDVRQU\VWUXFWXUHV VWUHQJWKHQHGZLWKVWHHOFODPSVDQGEROWVZKLFKLVEDVHGRQWKH ILQLWHGLVFUHWHHOHPHQWPHWKRG7KHGHYHORSHGPRGHOFDQEH XVHGIRUWKHVWLPDWLRQRIWKHVHLVPLFUHVLVWDQFHRIKLVWRULFDO GU\VWRQHPDVRQU\VWUXFWXUHVUHLQIRUFHGZLWKVWHHOFODPSVDQG VWHHOERWVZKLFKLVYHU\LPSRUWDQWIRUWKHVWUXFWXUHVFODVVLLHG DVFXOWXUDOKHULWDJH7KHPRGHOFDQDOVRKHOSWRPDNHWKHULJKW GHFLVLRQVUHJDUGLQJWKHUHVWRUDWLRQRIGU\VWRQHPDVRQU\ VWUXFWXUHVZKLFKKDYHH[SHULHQFHGGHWHULRUDWLRQRYHUWLPH

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7KLVZRUNKDVEHHQIXOO\VXSSRUWHGE\&RDWL DQ6FLHQFH)RXQGDWLRQ XQGHU WKH SURMHFW

*Development of numerical models for reinforced-concrete and stone masonry structures under seismic loading*EDVHGRQGLVFUHWHFUDFNV,3

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) 3DULVL 1 $XJHQWL ³(DUWKTXDNH GDPDJHV WR FXOWXUDO KHULWDJH FRQVWUXFWLRQV DQG VLPSOLILHG DVVHVVPHQW RI DUWZRUNV´ *Engineering Failure Analyses*YROSS 3%/RXUHQoR-*5RWVDQG-%ODDXZHQGUDDG³&RQWLQXXPRGHOIRU PDVRQU\ SDUDPHWHU HVWLPDWLRQ DQG YDOLGDWLRQ´ *Journal of Structural Engineering ASCE*YROSS /%HUWR$6DHWWD56RWWDDQG59LWDOLDQL³$QRUWKRWURSLF GPDJH PRGHO IRU PDVRQU\ VWUXFWXUHV´ *International Journal of Numerical Methods in Engineering*YROSS 6&DVROR³0DFURVFRSLFPRGHOOLQJRIVWUXFWXUHGPDWHULDOV5HODWLRQVKLS EHWZHHQRUWKRWURSLF &RVVHUDWFRQWLQXXP DQG ULJLG HOHPHQWV´ *International Journal of Solids and Structures* YRO SS

3$&XQGDOO³$FRPSXWHUPRGHOIRUVLPXODWLQJSURJUHVVLYH HODUJHVFDOH PRYHPHQWVLQEORFN\URFNV\VWHPV3XEOLVKHG&RQIHUHQFH3URFHHGLQJV VW\OH´LQ*Proc. of the Symposium on Rock Fracture (ISRM)* 1DQF\ )UDQFHSS 3$ &XQGDOO 5' +DUW ³1XPHULFDO PRGHOOLQJ RI GLVFRQWLQXD´ *Engineering Computations*YROSS & %DJJLR 3 7URYDOXVFL ³6WRQH DVVHPEOLHV XQGHU LQSODQH DFWLRQV FRPSDULVRQEHWZHHQQRQOLQHDUGLVFUHWHDSSURDFKHV´ *Computer Methods in Structural Masonry*YROSS & %DJJLR 3 7URYDOXVFL ³&ROODSVH EHKDYLRXU RI WKUHHGLPHQVLRQDO EULFNEORFN V\VWHPV XVLQJ QRQOLQHDU SURJUDPPLQJ´ *Structural Engineering and Mechanics*YROSS $1DSSL)7LQ/RL³$QXPHULFDOPRGHOIRUPDVRQU\LPSOHPHQWH GLQ WKHIUDPHZRUNRIDGLVFUHWHIRUPXODWLRQ´*Structural Engineering and Mechanics*YROSS SURJUDPPLQJ´*Computers & Structures*YROSS *Mechanics and Mining Sciences*YROSS GLPHQVLRQDO GLVWLQFW HOHPHQW PRGHO±3DUW ,, 0HFKDQLFDO FDOFXODWLRQV´ *International Journal of Rock Mechanics and Mining Sciences* YRO SS *Key Questions in Rock Mechanics*5RWWHUGDP%DONHPDSS '(0 DQG)(0'(0 PRGHOV LQ ' DQG '´ *Engineering Computations*YROSS *Engineering Structures*YROSS