Advancements in Wavelet-Based Structural Analysis Techniques

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Abstract—Time history dynamic analysis of structures is considered as an exact method while being computationally intensive. Filtration of earthquake strong ground motions applying wavelet transform is an approach towards reduction of computational efforts, particularly in optimization of structures against seismic effects. Wavelet transforms are categorized into continuum and discrete transforms. Since earthquake strong ground motion is a discrete function, the discrete wavelet transform is applied in the present paper. Wavelet transform reduces analysis time by filtration of non-effective frequencies of strong ground motion. Filtration process may be repeated several times while the approximation induces more errors. In this paper, strong ground motion of earthquake has been filtered once applying each wavelet. Strong ground motion of Northridge earthquake is filtered applying various wavelets and dynamic analysis of sampled shear and moment frames is implemented. The error, regarding application of each wavelet, is computed based on comparison of dynamic response of sampled structures with exact responses. Exact responses are computed by dynamic analysis of structures applying non-filtered strong ground motion.

Keywords—Wavelet transform, computational error, computational duration, strong ground motion data.

I. INTRODUCTION

TIME history analysis of structures is considered as an exact solution for seismic analysis of structures while intensive computational time is needed. Fourier Transform (FT) and Fast Fourier Transform (FFT) are methods, applied through time history dynamic analysis [1]. Strong ground motion of earthquakes are represented in time domain while FT and FFT denote the frequency domain of strong ground motion. Frequency content of strong ground motion is evaluated by frequency spectrum, computed by FT and FFT
[2].

Frequency domain is computed by FT. FTs apply sinus and cosine functions as fundamental waves which are applied from □□ to □□ , meanwhile there is one point as amplitude for each frequency. This difficulty will result in instability while applying non-harmonic waves. Therefore, wavelet transform is applied as a more powerful and effective approach towards frequency domain analysis of structures [2]. Wavelet transform has been applied in geophysics to analyze seismic data in view of oil and mine explorations. These transforms have been invented by mathematician about 20 years before their applications in signal processing. The pioneer researcher who applied wavelet transform in vibration analysis is Neylonde. Assessment of vibrated buildings by under-ground trains has been implemented by Neylonde [3]. Wavelet transforms have been applied in commercial data analysis of markets, discovery of cell membranes in biology, storage of 33 million fingerprints in Federal Bureau of Investigation, image processing, image exploration, animation control and image compaction and health monitoring in civil engineering
[4].

Wavelet transform can be applied to filter earthquake strong ground motions. This method will divide the earthquake records into two main parts, namely low and high frequencies. The contribution of high frequency portion of earthquake record in structural response is negligible. Therefore, the structure is excited by low frequency part of earthquake records. Application of wavelet transform to an earthquake record will reduce the record points by half then the computational time will be reduced proportionately.

Wavelets are mathematical functions which are applied towards signals decomposition into frequency components. Resolution of each component will be its scale factor. Wavelet transformation is mathematical convolution based on wavelet functions. Wavelet functions (which are entitled as infant wavelets) are forms of transformed and scaled of a function (namely parent wavelet) with finite length and severelydamped
[5].

FT represents the frequency content of a wave and does not show the time of occurrence for each frequency. Furthermore, application of harmonic functions as base waves in FTs causes instability while being applied for non-harmonic pulses. On the other hand, wavelet transforms are superior in comparison with FT in view of overcoming the mentioned shortcomings
[6].

The wavelet transform possesses a satisfactory characteristic of localization. For instance, one must compute many coefficients to achieve FT of a steep function, since the base functions for this transform are sine and cosine with constant coefficients. On the other hand, the energy of wavelet functions are localized while being quickly damped. Therefore, the wavelet transform has the superiority of compression in comparison with FT while a suitable parent wavelet is selected. In this research, the wavelet transform is applied to the strong ground motion of earthquake and even and odd data of earthquake records are applied the structure separately.

II. Wavelet Transform

Transformation and scaling of parent wavelets will produce wavelet functions. Scaling parameters which are applied to parent wavelet are computed as [7]:

$$g_{b,\tau}(t) = \frac{1}{\sqrt{b}}\, g\left(\frac{t-\tau}{b}\right), \quad b \in \mathbb{R},\ b \neq 0 \tag{1}$$

$g(t)$ is the parent function while b and τ are characterized as scale and transform factors, respectively. To give the impression of low frequency content of the signal, b value must be increased, while the parent wavelet will be stretched and will cover more part of signal. If b value is decreased then the parent function will be tightened up and the wavelet will show the lower frequency portion of the signal. τ Transforms the wavelet to the main signal and corresponds to time domain transformation [8].

Wavelets are real and imaginary functions in time and frequency domains and have analogous forms. To practice the wavelet analysis, the signal must be multiplied to the wavelet function and integration is calculated separately for each parts of sampled signal in time domain. The first step to calculate the wavelet transform of a sampled signal $x(t)$ is multiplication of the sampled signal to the wavelet function and integrate the result inside the signal's domain. The following equation represents the procedure [9].

$$WT(\tau, b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} x(t)\, g^{*}_{\tau,b}(t, \tau, b)\, dt \tag{2}$$

WT is the abbreviation of Wavelet Transform, $g_{b\tau}$ is calculated based on (1), and asterisk shows the imaginary conjugate. Scaling parameter in WT is similar to scale factor of a map. While the detailed parts are ignored in large-scale maps, similarly, large scale factor in WT corresponds to ignoring the detailed part of the signal and small factor corresponds to display the detained parts of the signal. Likewise, in frequency domain, lower frequency (larger scale) corresponds to general characteristics of the sampled wave, while higher frequencies represent the detailed and local characteristics. While the higher frequencies occur in a low duration, the lower frequencies are present in much more duration of the sampled signal [10].

If the wavelet possesses average value of zero and parent function amplitude is rapidly damped, then this convolution can be considered as locally affected transformation. Under these conditions, the transformation is reversible. The inverse wavelet transformation can be calculated by [11]:

$$x(t) = \frac{1}{c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} WT(\tau, b)\, b^{-2}\, g_{\tau,b}\, d\tau\, db \tag{3}$$

III. Wavelet Transform in MATLAB

Earthquake records are discrete signals for which the discrete wavelet transformation is applied in the present research. For instance, applying *Harr* wavelet transform can be applied by the following equation based on MATLAB.

$$[A, D] = dwt(Earthquake, 'haar') \tag{4}$$

A indicates the lower frequency part of the earthquake wave which must be applied to the structure and Dshows the higher frequency part and is not applied through dynamic analysis of structures. According to the function of transformation, wavelet transforms can be categorized to *Harr*, *Dmey*, *Sym1*, *Coif1* etc. Applying wavelet transform will double earthquake time step.

Displacement matrix, which represents time history of seismic response, is derived by time history analysis of sampled structure. Each row of this matrix corresponds to displacement of one degree of freedom at each time step. Finally, real displacement of the considered DOFs is calculated through inverse wavelet transform of the corresponding rows. To achieve real displacement, inverse wavelet transform must be applied to each row of displacement matrix. Certainly, the inverse formula must be selected based on wavelet transform which has been applied in the first step of analysis. The following equation illustrates inverse wavelet analysis for the first row of displacement matrix.

$$NewDisp(1,:) = idwt(Displacement(1,:),[], 'haar') \tag{5}$$

IV. Applied Earthquakes and Wavelet Functions

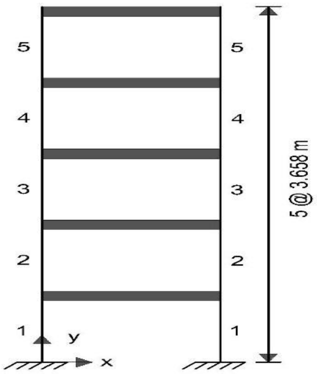
In this paper, north-south component of Northridge earthquake is selected as strong ground motion affected to the structures. Eight wavelet functions, namely *Haar*, *Sym2*, *Sym3*, *Sym4*, *Sym5*, *Db1*, *Db2* and *Db3* are considered as transformation formulas.

V. First Case Study

Five-story shear buildings with one span are considered as case studies in this paper (see Fig. 1). Rigid diaphragms assumption is considered which result in rigid beams and weight of each floor is considered as 21 tons. Applied cross sections for each frame are shown in Table I and the sampled shear structures are presented in Table II.

Table III represents the approximation error for prediction of roof displacement while filtered ground motion is applied to the structure. This approximate value is compared with exact response, which is calculated through time history dynamic analysis of sampled structure excited by unfiltered ground motion.

Figs. 2 and 3 show exact displacement time history of shear frame NO. 1 in 10 second and comparison with *Haar* and *Db3* wavelet transforms, respectively.



frame

Fig. 1 A five story shear building

No.	Profile
1	Box 180*180*16
2	Box 220*220*17.5
3	Box 240*240*20
4	Box 260*260*20
5	Box 280*280*20
6	Box 300*300*20 7
	Box 320*320*20
8	Box 340*340*20

TABLE I
AVAILABLE PROFILES FOR THE ALL CASE STUDY

TABLE II
SHEAR FRAME STRUCTURES AND SECTIONS USED IN THEM

Structure no.	Element groups no.

TABLE III
DISPLACEMENT ERROR PERCENT FOR THE ROOF OF SHEAR FRAME

Structure no. and time (s)	Wavelet				
	Haar or Db1	Sym2 or Db2	Sym3 or Db3	Sym4	Sym5
Time	0.83	0.26	0.36	0.33	0.36
Time	0.320	0.351	0.344	0.351	0.322
Time	0.68	0.14	0.02	0.12	0.07
	0.323	0.344	0.361	0.322	0.352
	0.41	0.60	0.20	0.07	0.10
	0.348	0.338	0.357	0.342	0.314

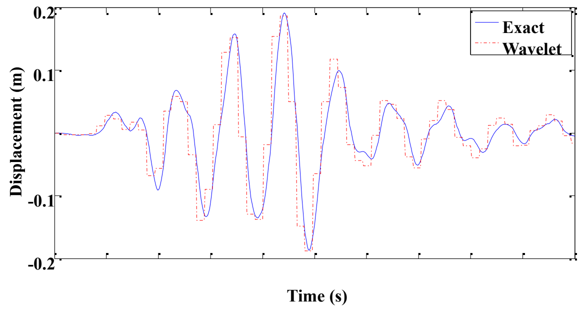
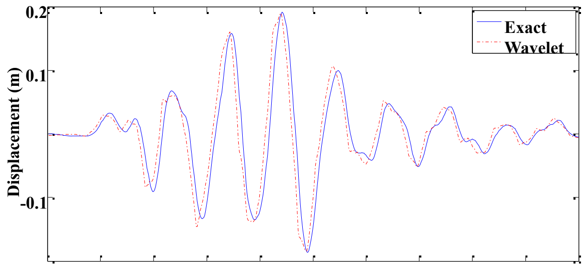


Fig. 2 Roof displacement derived by Haar wavelet compared with exact ones for first shear



0 1 2 3 4 5 6 7 8 9 10
Time (s)

Fig. 3 Roof displacement derived by Db3 wavelet compared with exact ones for first shear frame

Figs. 4 and 5 show exact displacement time history of shear frame NO. 5 in 10 second and comparison with *Haar* and *Db3* wavelet transforms, respectively.

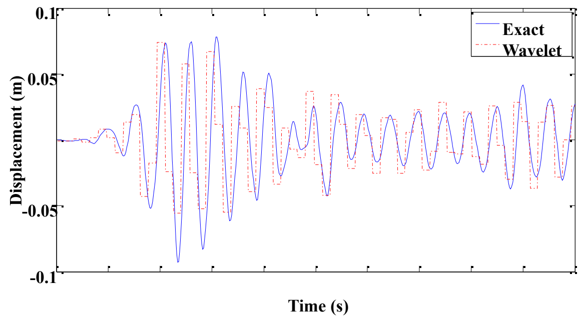


Fig. 4 Roof displacement derived by Haar wavelet compared with exact ones for fifth shear frame

4	0.37	1.49	0.32	0.07	0.14
Time	0.359	0.375	0.345	0.62	0.330
5	1.45	0.04	0.06	0.85	0.55
Time	0.335	0.345	0.325	0.321	0.317
Average error	0.74	0.50	0.19	0.28	0.24
Average time (s)	0.337	0.350	0.346	0.339	0.327

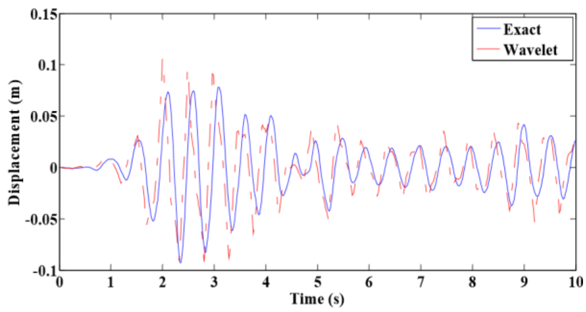


Fig. 5 Roof displacement derived by Db3 wavelet compared with exact ones for fifth shear frame

VI. SECOND CASE STUDY

As the second case study, one-span five-story moment frames are considered (see Fig. 6). A uniform service load of 1000 kg/m is applied to the structural beams additional to the uniform weight of the elements.

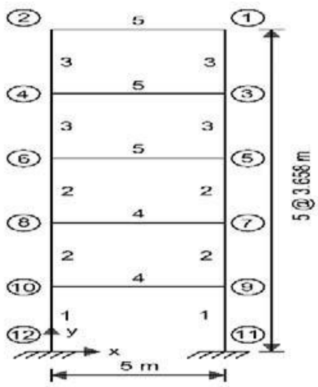


Fig. 6 A Five-story bending frame

TABLE IV
FIVE-STORY BENDING FRAME STRUCTURES AND SECTIONS USED IN THEM

Structure no.	Element groups no.

TABLE V
ERROR PERCENT OF MAXIMUM ROOF DISPLACEMENT FOR FIVE-STORY BENDING FRAME

Structure no. and time (s)	Wavelet Haar or Db1	Sym2 or Db2	Sym3 or Db3	Sym4	Sym5
	0.27	0.81	0.08	0.02	0.01
Time	1.858	1.873	1.861	1.875	1.886
	1.13	0.48	0.24	0.22	0.30
Time	1.874	1.868	1.873	1.861	1.874
	0.43	0.91	0.07	0.10	0.05
Time	1.854	1.901	1.905	1.902	1.873
	0.57	1.50	0.20	0.02	0.11
Time	1.879	1.889	1.873	1.887	1.875
	3.53	2.07	0.27	0.88	0.59
Time	1.878	1.883	1.885	1.892	1.892
Average error	1.18	1.15	0.17	0.24	0.21

Average time (s)	1.868	1.882	1.879	1.883	1.880

In comparison with shear buildings, similar cross sections are considered in sampled moment frames. Table IV shows five considered moment frames.

The error of prediction for roof displacement of the sampled moment frames is presented in Table V. In approximate analysis filtered strong ground motion are applied to the structure while exact solution is derived based on full time history dynamic analysis of sampled moment frames applying unfiltered strong ground motion.

Figs. 7 and 8 show exact displacement time history of bending frame NO. 1 in 10 second and comparison with *Haar* and *Db3* wavelet transforms, respectively.

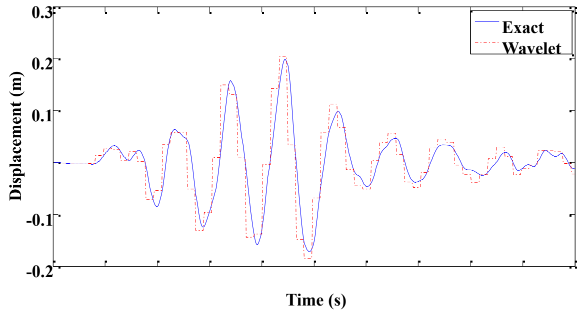


Fig. 7 Displacement of joint 1 in X direction by Haar wavelet compared with exact ones for first bending frame

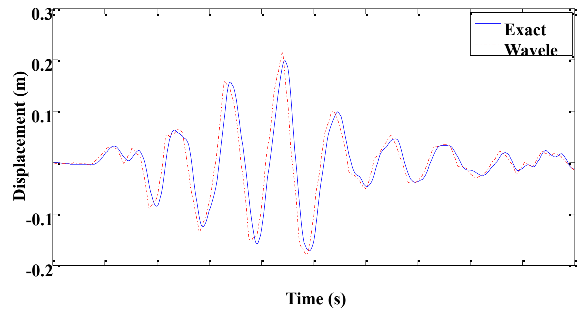


Fig. 8 Displacement of joint 1 in X direction by Db3 wavelet compared with exact ones for first bending frame

Figs. 9 and 10 show exact displacement time history of bending frame NO. 5 in 10 second and comparison with *Haar* and *Db3* wavelet transforms, respectively.

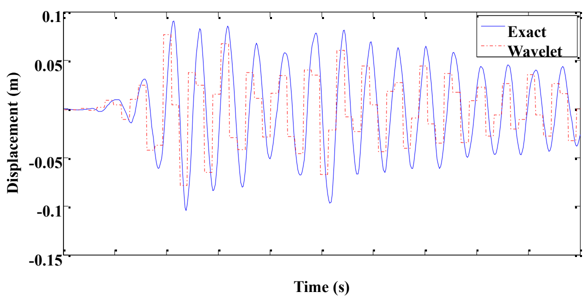


Fig. 9 Displacement of joint 1 in X direction by Haar wavelet compared with exact ones for fifth bending frame

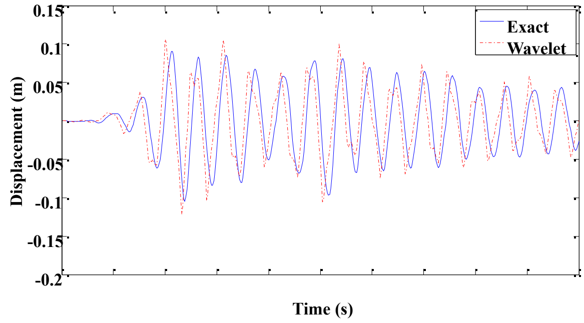


Fig. 10 Displacement of joint 1 in X direction by Db3 wavelet compared with exact ones for fifth bending frame

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