

Optimization of Seismic Design Parameters for Maximum Expected Loss Reduction

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Abstract—An approach to compute optimum seismic design parameters is presented. It is based on the optimization of the expected present value of the total cost, which includes the initial cost of structures as well as the cost due to earthquakes. Different types of seismicity models are considered, including one for characteristic earthquakes. Uncertainties are included in some variables to observe the influence on optimum values. Optimum seismic design coefficients are computed for three different structural types representing high, medium and low rise buildings, located near and far from the seismic sources. Ordinary and important structures are considered in the analysis. The results of optimum values show an important influence of seismicity models as well as of uncertainties on the variables.

Keywords—Importance factors, optimum parameters, seismic losses, seismic risk, total cost.

I. INTRODUCTION

THE optimum decision process for structural systems to be built on seismic sites can be made by selecting a combination of seismic design criteria, quality control, and repair and maintenance strategies leading to the minimum present value of the sum of the initial costs and those that may occur during the life cycle of the system. In the latter, those costs due to possible damage and failure as well as actions of repair and maintenance are included. If the relationship between utility for society and expected present value of its assets is taken as linear, a design will be approximately optimum when it minimizes the objective function given by initial cost and expected present value of the losses due to earthquakes [1], [2]. This approach does not take into account higher order statistical moments of monetary values, risk attitudes, and cognitive limitation of decision-makers. Furthermore, the economic model does not include the design cost as well as all studies required by this design. It is advisable to use a decision tree, which shows alternatively through branches, all the decisions that the designer can make and all the events that can occur [23]. This diagram allows for analyzing as a whole all possible solutions because probabilities and utilities can be set in the appropriate places [3]. Other decision rules can be used to identify the optimum seismic design such as the stochastic dominance, which includes the use of restrictions in the quality of social life, the socially tolerable risk, and attitudes toward risk of the decision-maker. The cumulative prospect theory developed by [4] includes several aspects of human cognitive process and

risk perception in decision making. As long as these formulations are still at the research stage, the life cycle optimization described

at the beginning of the paragraph is used here in order to find the optimum parameters.

This paper starts by describing three different seismicity models, including one for characteristic earthquakes, which are usually used to represent the occurrence of earthquakes. Because of the roles played by costs in the decision process, we then discuss initial costs as well as losses due to earthquakes. Then the effect on optimum values due to uncertainties in the parameters is examined. Different cases for optimum design coefficients are analyzed, and importance factors are computed for near-source sites as well as for those far from the seismic source.

II. SEISMICITY

A. Local Seismicity, Poisson Process with Known Parameters

Seismic activity is usually well represented by curves like those shown in Fig. 1 where the exceedance rate, λ , is the number of earthquakes per unit volume and per unit time having magnitudes greater than M . Up to a few years ago, it was assumed that the magnitude-recurrence curve for a local seismic source had the shape of a straight line like curve A in Fig. 1, as a result of the analysis of observed data in the whole earth's crust or in large zones. Gutenberg and Richter [5] obtained expressions which results can be written as:

(1)

where a and b are constants. On the other hand, according to Cornell and Vanmarcke [6] the exceedance rate of magnitudes of the earthquakes originated in a tectonic province can be taken as λ , where c ,

d , and e are unknown parameters, and M_0 is the magnitude above which the seismic catalogue is complete. This function can be conveniently expressed by:

(2)

This curve is a straight line for small earthquakes, and as M increases, it turns concave downwards taking a value of zero for $M = M_0$, and accepts the fact that M_0 is the maximum magnitude that can be generated in the corresponding seismic source (curve B in Fig. 1).

In the process of occurrence of earthquakes discussed above, a hypothesis is made that the probabilistic distribution

of waiting times between seismic events, with magnitudes in a given interval, is of the exponential type. Thus, the number of events with magnitudes in this interval has a Poisson distribution, that is, the hazard does not change with the time elapsed without the occurrence of large earthquakes. However, certain discrepancies with this model have been recognized because statistical data show that in some regions, the relationship between frequency and magnitude of earthquake occurrence presents anomalies consisting of the lack of earthquakes of certain magnitudes compared to the worldwide average. That is, earthquake magnitudes are sometimes grouped within a narrow band of values, giving rise to the so-called characteristic earthquake [7], [8]. This is why it has been concluded that seismicity models should represent seismic activity as the superposition of two subprocesses (curve C in Fig. 1). In the first subprocess, events occur completely in a random manner, without it being possible to make some prediction either deterministic or semideterministic. In our case, this subprocess will be given by (2). The second subprocess consisting entirely of characteristic earthquakes, with large magnitude whose intervals between occurrences are less uncertain than those associated to the first subprocess, can be put in the form

(3)

where λ is a constant. Thus, the total local seismicity is given by adding (2) and (3).

(4)

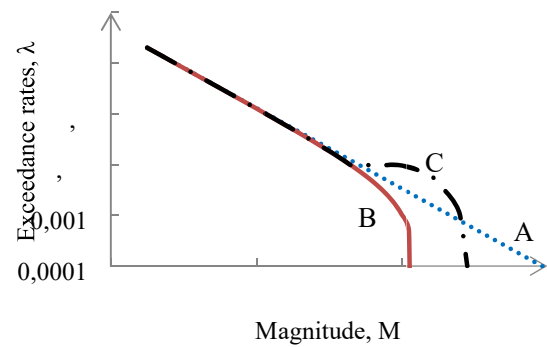


Fig. 1 Seismicity models

B. Attenuation

Attenuation laws are relations among magnitude, distance and intensity in firm ground, the majority of expressions provide peak ground acceleration, velocity, or displacement, as well as response spectral ordinates for a given period and critical damping. Under linear conditions and for the far field, peak ground acceleration can be represented by [9]

(5)

where α is a function of the coordinates of the focus or rupture area and of the site of interest, as well as of the properties of the material beneath the site, and β is a constant.

C. Regional Seismicity

The exceedance rate of z can be calculated by combining the exceedance rates of magnitudes and the attenuation law. This can be done for the different seismicity models; Here we show the one corresponding to the two subprocesses. Thus, by combining (4) and (5), we obtain the exceedance rate of z as:

(6)

which is valid when the material of earth's crust behaves linearly between the source and site of interest. Furthermore, the site-source distance is large compared to the dimensions of the rupture area. α and β correspond to α and β respectively, and λ and λ_0 are constants. For convenience, we will write λ . Fig. 2 shows the

magnitudes of characteristic earthquakes conforms to a slip- 0.015 if 1, and 0.188-117.8' if predictable model [14], [15]. If t denotes the time of the last 1 (see Fig. 3). characteristic earthquake, we can write

(8)

exceedance rates of intensities for the three different cases considered here.

Now, let λ denote the density of occurrence of earthquakes with intensity I . Thus, we could write:

(7)

where δ is Dirac's delta. Here λ represents the occurrence rate of characteristic earthquakes.

Equation (5) still works for small I near a source, but there is a saturation phenomenon for large I . Thus, for large earthquakes near a source, λ does not increase in the same proportion with I as it does for large distances [10]. Something similar occurs at sites distant from the source, when the nonlinear behavior of the soil reduces the response spectral ordinates for large magnitudes [11]. We assume in this case that at a given exceedance rate, all values of I duplicate except I_{max} , which is the maximum intensity that can occur at the site of interest.

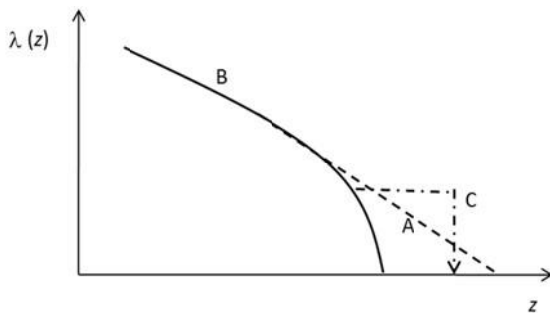


Fig. 2 Exceedance rates of intensities

D. Magnitudes and Occurrence Times of Characteristic Earthquakes

Based on the analysis of self-organizing systems [12] and on data from real earthquakes [13], it can be assumed that the

For Mexican subduction earthquakes, the threshold magnitude of the characteristic earthquake, the corresponding recurrence time, and the constant are [13]: 7.4, 26.7, and 1.43, respectively.

The assumption that arrival times of all earthquakes at the site of interest constitute a Poisson process is adequate when nothing is known about arrival times other than the magnitude exceedance rates, or when significant earthquakes can arrive from a number of independent sources. However, when significant earthquakes originate in a single source and there is an idea of the recurrence period of the characteristic earthquake, one should take into account the non-Poisson nature of their arrival times. Jara and Rosenblueth [13], based on a study of Mexican characteristic subduction earthquakes, find that the best probability density function to describe the occurrence of large earthquakes is the lognormal distribution.

III. COSTS

A. Initial Construction Cost

Let u be the initial cost of a structure designed with coefficient c . Based on work by [16]-[20], it is reasonable to adopt

(9)

where, if the structure is not designed to resist earthquakes, u_0 would be its corresponding cost and R_0 would be its lateral resistance, and α takes values of 0.05 to 0.13 for high-rise and low-rise buildings, and β are constants with values of 0.5, and 1.1 to 1.4 for low-rise and high-rise buildings, respectively.

B. Losses Due to Earthquakes

Direct Material Loss

Let L be the direct material loss due to damage to the building itself when subjected to an intensity I . According to data and studies done by [21], [22], given an earthquake of intensity I , the expected loss due to material damage to the building itself at the instant of the earthquake is proportional to the power 1.6 of the quotient I/c when $I/c < 1$. We will take $L = c \cdot (I/c)^{1.6}$, where the function must increase with I , thereby decreasing as c increases so that $\lim_{c \rightarrow 0} L = 0$ and $\lim_{c \rightarrow 1} L = 1$. Furthermore, it must tend very fast to zero when I tends to zero because we know that earthquakes of low intensity do not cause any damage. Thus according to empirical data and all considerations made, the following expressions are used for L , $L = c \cdot (I/c)^{1.6}$ when $I/c < 1$ and $L = I^{1.6}$ when $I/c > 1$.

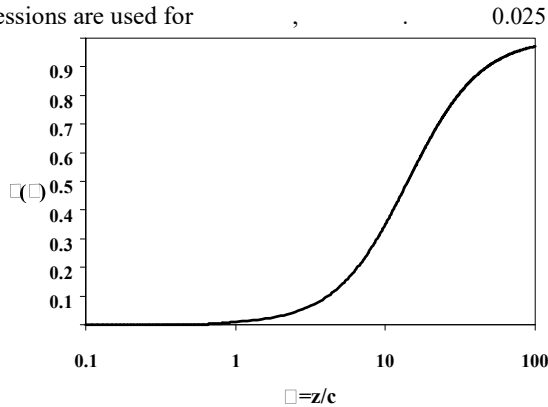


Fig. 3 Loss rate of structures in terms of intensity and seismic design coefficient

Indirect Economic and Non-Economic Loss

This loss represents all damages that earthquakes cause to society. It must be insignificant when I/c is small, because there is practically no damage done to the contents of the buildings. Furthermore, it should exceed L when I/c is close to one, because it corresponds to buildings under collapse, causing usually nearly the total loss of its content, the loss of many human lives and the economic chaos in the affected area.

In computing all possible losses, intangibles such as human lives must be taken into account. In this case, it is not a trivial matter to establish monetary equivalents, and this kind of purely economic approach deserves further study, because just considering this loss as an additive term in the formation of an objective function may lead to absurd results.

Different approaches have been developed to deal with the problem, namely, human capital, consumption and its variations, consideration of legacies or bequests, willingness to pay, and quality of life. A review of these methods is done by García-Pérez [23], and a lower limit is obtained for this intangible by making it equal to the expected present value of the person's contribution to the gross domestic product. By using data for Mexico, this limit results in 45 000 US dollars. The main objection to this human-capital approach is that it looks only at the economic side of the problem. Mishan [24] suggested that in resource allocation, in order to achieve an improvement in the sense of Pareto, it is required to take into account each person's willingness to reduce his/her risk of dying. A Pareto improvement is said to exist when individuals, who gain from a social change, are able to compensate those who stand to lose from the change and still

leave a net gain. Also, Usher [25] published a formal treatment to the problem of establishing the amount that a rational person must be willing to invest, in order to reduce such a risk, taking into account his/her utility curve. We should, therefore, look at the amount that a person is willing to invest in order to reduce the probability of losing her/his life. García-Pérez [23] discusses the willingness-to-pay approach and computes a factor, using an individual utility curve, whereby one has to multiply the value assigned to life by the human-capital approach and obtain the value that the person would assign to her/his life. This factor is always greater than one and could be much greater. Research is needed especially regarding the choice of utility curves in both individual and social problems.

Expected Present Value of Seismic Losses

The loss caused by an earthquake of intensity I at the instant that it occurs, $L(I)$, must include all seismic losses given by the direct material loss and the indirect economic and noneconomic loss as discussed above. Thus $L(I) = L + L_{ind}$, where L_{ind} is a factor considerably greater than 1.

If the earthquake arrival times constitute a multiple Poisson process, and we assume that the original condition is restored to the structure after each earthquake, and the discount rate r is independent of time and the expected cost of damage and failure per unit time is $\lambda L(I)$, [26], then the expected present value of all seismic losses becomes $\lambda L(I) / (r - \lambda)$, and after substituting all variables, the following expression is obtained:

$$-\int_0^{\infty} \frac{L(I) \lambda e^{-\lambda t}}{r - \lambda} dt \quad (10)$$

It is convenient to write $L(I)$ in (10) and integrate with respect to I rather than with respect to t . Thus, we get:

$$-\int_0^{\infty} \frac{L(I) \lambda}{r - \lambda} dI \quad (11)$$

Similar expressions can be derived considering L_{ind} by using the corresponding numerical value of L_{ind} , the maximum values of the intensity, and excluding characteristic earthquakes. This will be illustrated through some examples below.

C. Expected Present Value of the Total Cost

The expression to be minimized is the expected present value of the total cost including the initial cost (9), as well as the losses due to earthquakes (11), given by:

$$-\left\{ \gamma + \int_0^{\infty} \frac{L(I) \lambda}{r - \lambda} dI \right\} \quad (12)$$

IV. UNCERTAINTIES

So far, we have treated all parameters as deterministic. However, uncertainties in each one of them become very high. Thus, we now take into account the effect on spectral ordinates of uncertainties in some parameters. We treat r , λ , and the initial cost of a structure as deterministic, since r , the base shear coefficient, is chosen by the designer or fixed by a code. Since most

parameters are obtained from linear regression between their logarithms and known quantities, we assign lognormal distributions to random variables with standard deviations and modes or deterministic values in (11). Uncertainties in the structural capacity are considered by assigning a standard deviation of 0.4 to the design coefficient, which is reasonable for reinforced concrete frames. The expected value of a linear function of a power of a random function, for example, μ^x , where μ is any real number, is computed as the function's median times $\exp(x/2)$. In the case of nonlinear functions, the two-point estimates method developed by Rosenblueth [27] is used. We also take 12, 1, 0.02, 0.2, the mode of μ , and μ/σ and so on. Thus, the expected present value of the total cost with uncertainties is:

$$C = \sum_{i=1}^n C_i \quad (13)$$

where, $C_i = \mu_i \exp(\frac{\sigma_i^2}{2})$. And for $i=1,2$ we have that: $\mu_1 = 19.78 / 0.01$, etc.; $\mu_2 = 0.1978$, and $\sigma_2 = 0.1978$.

V. DISCOUNT AND CONSTRUCTION RATES

The present values of the losses have been obtained by considering a discount function, $\exp(-\gamma t)$, where γ is a constant discount rate, often taken as 0.05/yr, because this is the value used in major financial transactions carried out in recent decades. However, surveys in the US of the discount rate [28], which must be applied to the social value of a human life, lead to the conclusion that γ decreases rapidly with time. Any discount function can be approximated as closely as wished by replacing it with $\sum_i \rho_i \exp(-\gamma_i t)$ where $\sum_i \rho_i = 1$ and

$\rho_i > 0$ for all i . Whatever the parameters ρ_i and γ_i may be, if the process under study is Poisson, there is always an equivalent discount rate independent of time that leads us exactly to the same results for the total expected present value [29]. By using an expression of the form $\rho_1 \exp(-\gamma_1 t) + \rho_2 \exp(-\gamma_2 t)$, Rosenblueth [29] finds an equivalent discount rate of 0.0686.

In this study, we have been dealing with a single building that we assume will be designed and built immediately. Codes are intended to be applied to buildings that will be erected at different times and over several years, for example t_1, t_2, \dots, t_n . In this case, it is convenient to minimize the expected present value of all costs of the structures that will be built in the zone where the codes apply. Let N denote the expected number of structures to be built per unit area and per unit time. The expected present value of the number of buildings that will be built is then $N \sum_{i=1}^n \exp(-\gamma_i t_i)$. The expected present value of the initial costs is $C_0 \sum_{i=1}^n \exp(-\gamma_i t_i)$. Thus if a building is constructed at time t after the code is enacted, and if the discount rate is constant, then the expected seismic loss for this building actualized to time t is given by (11). Now the number of buildings constructed between t and $t+dt$ is $N dt$. Therefore, the expected present value of the losses is [30].

The problem of finding optimum seismic design parameters, when different structural types are built in a region, has been solved by using both genetic algorithms [31] and artificial neural networks [32].

If we are interested in a single structure built at 0, we find affected by the factor $1/\gamma$ which, in the case of the equivalent discount rate gives a value of 14.6. Now, if we are concerned with a building code that will be in use for ten years before it is updated, and if γ is constant over this period, we find that by using γ , must be multiplied by 5 and that the factor in (13) is 116. Therefore, C has a weight relative to C_0 , which is 1.6 times greater than that in the single structure when we consider that buildings will be constructed over the ten year period.

VI. TIME DEPENDENT NON-POISSON CHARACTERISTIC EARTHQUAKES

Based on a study by Jara and Rosenblueth [13], we can assume for illustrative purposes that characteristic earthquakes belong to either of two populations. In the first population, twenty per cent of the events have an exponential distribution for the time with expected value of 1.5 yr between events, and the second population has a lognormal distribution with median 40.6 yr and standard deviation of natural logarithm of equal to 0.4. The expected value of t in the second population is then $40.6 \exp(0.4/2) = 43.7$ and that which is for all characteristic events is 36.7. In the case of a single structure to be designed and built immediately, if the slip-predictable process is ignored, by numerical integration it is found that the expected loss at the time that a characteristic earthquake strikes must be multiplied by a factor that varies between 0.18 and 0.41 corresponding to 5 and 75 yr after the last earthquake, rather than be multiplied by $1/0.4$, as in a Poisson process to obtain the expected present value of all such earthquakes.

We use the data from subduction earthquakes from the coast of Mexico given by (5), and we will take the maximum value of equal to 8. Then it is found that the increase in C with time increases the lower limit by a small factor while the upper stays below 0.41 $\exp(8/7.4)$ that turns to be 0.84 when using 1.2 [9]. If we are concerned with a building code that will be in use for ten years before it is updated, and if we assume that γ is time independent, it is found that the lower limit exceeds $1.6 \times 0.18 = 0.288$ while the upper limit is less than $1.6 \times 0.84 = 1.344$, regardless of when the last characteristic earthquake occurred.

VII. EXAMPLES

A. Optimum Seismic Design Coefficients

Far-Field Site

Three different types of structures will be under study representing high- medium- and low-rise buildings. The corresponding parameters used in (9) are shown in Table I. The following values are used in the calculations:

$\gamma = 0.05$, $\sigma = 0.4$, $\mu = 3.3$, 0.05, 0.4 and 0.8, both with 0.5. Optimum values of C are obtained by minimizing the expected present value of all costs. This minimization process requires that $\partial C / \partial \mu = 0$. Thus, we obtain optimum values for the three different types

of structures under study for both deterministic parameters and with uncertainties and for three local seismicity curves as shown in Table II. This table displays results in column A considering the Gutenberg and Richter curve, those corresponding to Cornell and Vanmarcke in column B, including characteristic earthquakes (12) in column C. The results considering uncertainties with (13) are presented in the last column of this table.

TABLE I
PARAMETERS FOR THREE DIFFERENT TYPES OF STRUCTURES

Parameters	Type of structure		
	High	Medium	Low
	0.05	0.1	0.13
	0.5	0.5	0.5
	1.4	1.2	1.1

TABLE II
OPTIMUM SEISMIC DESIGN COEFFICIENTS FOR FAR-FIELD-SITE

Structural type	Using (12) (W)			Using (13) (\bar{w})
	A	B	C	
High	0.166	0.174	0.341	0.525
Medium	0.151	0.155	0.316	0.489
Low	0.140	0.147	0.296	0.335

Near-Field Site

Consider now a site in the near-field. Equation (5) still works for small λ , but is no longer valid for large magnitude earthquakes due to a saturation phenomenon. Thus, for large earthquakes near a source, does not increase in the same proportion with λ as it does for large distances [10]. The following intensities are used in order to calculate the optimum coefficients, given by the ordinates of the pseudoacceleration spectrum expressed in terms of the gravity acceleration, 0.5, 0.9 and 20 g . Results for the three different types of structures are displayed in Table III.

Importance factors at a near-field can be computed following a methodology developed by García-Pérez et al [33]. Usually building codes require that very important structures (those whose failure or collapse might cause a large loss of lives, an extraordinary economic loss, and public buildings that are essential during emergencies) be designed for a seismic coefficient equal to that used for ordinary structures multiplied by the importance factor. Therefore, we find the value of c first, which at the far-field minimizes the total cost. This c applies to ordinary structures taken from Table II column C, because we will consider the two seismicity subprocesses. After this, we compute the factor by which we must multiply b to increase the computed optimum to 1.5 (important structures), so that the importance factor is equal to 1.5 at the far-field site. Now we go to the near-field site and compute the optimum design coefficients, assuming that the values of c for both ordinary and important structures are the same as at the far-field site. These values are given in Table III. The ratio between these values gives us the importance factor corresponding to the near-field site as displayed in Table III. This

factor decreases for the near field site as was previously pointed out in [33].

TABLE III
OPTIMUM SEISMIC DESIGN COEFFICIENTS FOR NEAR-FIELD-SITE

Structural type	Ordinary	Important	Importance factor
High	0.425	0.570	1.34
Medium	0.401	0.536	1.34
Low	0.377	0.508	1.35

VIII. CONCLUDING REMARKS

The expected present value of total cost is used to compute optimum seismic design parameters for sites far and near a seismic source, respectively. High, medium and low structural types have been considered in the analysis. Different seismicity models are used and uncertainties are included in some variables to study their influence in the computation of optimum values. Concepts such as discount factor, construction rate, and indirect economic and non-economic loss are reviewed. The results show that taking into account the concepts studied here modify the optimum values, and that importance factors are lower in the near-field site assuming that the importance factor at a far-field site is optimum.

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