

# Mathematical Modeling of Complex Systems

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**Abstract** 7KLV VWXG\ LQYHVWLJDWHV WKH HIIHFV RI WZR LPSRUWDQW SDUDPHWHUV VOHQJWK RI FXULQJ SHULRG DQG SHUFHQW DJH RI WKH DGGHG ELQGHU RQ WKH VWUHQJWK RI VRLO WUHDWHG ZLWK 23& \$Q LQWHUPHGLDWH SODVWFLW\VLW\FOD\H\VRLOZLWKPGLXPRUJDQLF FRQWHQWZDVXVHGLQ WKLVVWXG\7KLVVRIWVRLOZDVWUHDWHGZLWKGLIHH UHQWVSHUFHQW DJHVRID FRPPHUFDOO\DYDLODEOHFHPHQW\SH1ODERUDWR UH[SHULPHQWV ZHUHFDUULHGRXWRQWKHVRLOWUHDWHGZLWKDQG2 3& EWKHGU\ZHLJKWWRGHWHUPLQHWHKHHIHFVRI23&RQ WKHFRPSDFWLRQ SDUDPHWHUV FRQVLVWHQF\ OLPLWV DQG WKH FRPSUHVVLYH VWUHQJWK 8QFRQILQH FRPSUHVVLYH VWUHQJWK8&6 WHVW ZDV FDUULHG RXW RQ FHPHQWUHDWHG VSHFLPHQV DIWHU H[SRVLQJ WKHP WR GLIHHUHQV FXULQJ SHULRGV DQG D\7KHUHVXOWVRI8&6WHVWZHUH XVHG WR GHYHORS D QRQOLQHDU PXOWLUHJUHVVLQRQ PRGHO WR ILQG WKH UHODWLRQV KLS EHWZHHQ WKH SUHGLFWHG DQG WKH PHDVXUHG PD[LXP FRPSUHVVLYH VWUHQJWK RI WKH WUHDWHG VRLOTX7KH UHVXOWV LQGLFDWHG WKDW WKHUH ZDV D VLQJLILFDQW LPSURYH PHQWLQWKH LQGH[ RI SODVWFLW\ ,3E\WUHDWLQJZLWK23&3ZDVGHFUHDVHG IURPWRE\ XVLQJRI23&WKLVSHUFHQW DJH ZDVHQRXJKWRLQFUHDVH WKH8&6 RI WKH WUHDWHG VRLO XS WR N3D DIWHU GD\V RI FXULQJ :LWK UHVSHFWWRWKHVWDLVWLFDO PRGHO RIWKH SUHGLFWHG FWHGTXWKHUHVXOWVVKRZHG WKDW WKH UHJUHVVLQRQ FRHIILFLHQWV5 ZDV HTXDO WR ZKLFK LQGLFDWHV DJRRGUSURGXFLELOLW\IRUWKHFRQVW UXFWHGPRGHO

**Keywords** 2&HHPHQW DGPL[WXUHVRIWVRLO VWDELOLVDWLRQ JHRWHFKQLFDO SDUDPHWHUV XQFRQILQH FRPSUHVVLYH VWUHQJWK PXOWL UHJUHVVLQRQPRGHO

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8(WRWKHUDSLGGHYHORS PHQWR IXUEDQLJWLRQW KHUHLVD PRVWDFHSHWGHGPHWKRGRWRPLWLJDWHWKHXQZDQ WHGVRLOSURSHUWLHV

+0-DIHU3RVWJUDGXDW5HVHDFK6WXGHQW\LYHUSRRO-RKQ0RRUHV 8QLYHU\VLW\6FKRRRIWKH%XLOW(QYLURQPHQW+HLVDQVRZ LWKWKH'HSDUWPHQW RI&LYLO(QJLQHHULQJ)DFXOW\RI(QJLQHHULQJ8QLYHU\VLW\RI %DE\ORQ,UDTH PDLO+0-DIHU#OMPXDFXN .KDOLG6+DVKLPDQG\$OL:\$SDWWEDELDUH3K'VWXGHQWLQ WKH6FKRRORI 7KH%XLOW(QYLURQPHQW\LYHUSRRO-RKQ0RRUHV8QLYHU\VLW\8.7KH'DUHDQVRZ LWKWKH'HSDUWPHQWRI&LYLO(QJLQHHULQJ)DFXOW\RI(QJL QHHULQJ8QLYHU\VLW\RI%DE\ORQ,UDT 'U:\$WKHUWRQ%(QJ+RQV3K')+(\$3URJUDPPH\HDGHULVZLWKW KH 'HSDUWPHQWRI&LYLO(QJLQHHULQJ\LYHUSRRO-RKQ0RRUH V8QLYHU\VLW\3HWHU -RVW(QWHUSULVH&HQWUH%XURP6WUHHW\LYHUSRRO/\$)8. &HHPHQW VWDELOLVDWLRQ LV WKH FODVVLFDQ PHWKRQ IRU FKHPFDQ VWDELOLVDWLRQ ZKLFK LV DWWULEXWHG WR WKH DJORPHUDWLRQ DQG IORFFXODWLRQSKHQRPHQRQW DNHSDPHZLWKLQ WKHVRLOVWUXFWXUH IRU ERWK RI VKRUW DQG ORQJ WHUP DORQJ ZLWK WKH K'GUDWLRQ UHDFWLYLW\KDSHQVDPHQJWKH ZDVHUFHPHQW DQGVRLOS DUWLFOHV 7KH K'GUDWLRQ UHDFWLRQ SURGXFW FHPHQWLWLRXV FRPSRXQGV ZKLFK DUH FDOOHG HLWKHU FDOFLXPVLOLFDWHK\GUDWHG&6+ RU FDOFLXPDOXPLQLXPK\GUDWHG&\$+ GHSHQDQW RQ WKH DYDLODEOH\SHRISR]]RODQLFFRPSRXQGLQWKH UHDFWVRLDQDQ WKHVHFHPHQWLWLRXVFRPSRXQGV DUHUHVSRQVL EOHWRLPSURYHVRLO SURSHUWLHV DQGLQFUHDVH WKHVRLOVWUHQJWK >@+RZHYHUFHPHQW VWDELOLVDWLRQ WHFKQLTXH LV YHU\ FRPSOH[ DQG LW FDQ EH LQIOXHQFHGE\VHYHUDOVSHFWVXFKDVWKHH[L VWHQFHRIRUHLQJ PDWHULDOV VXFK DV WKH RUJDQLF PDWHU WKH DPRXQW RI QDWXUDO PRLVWXUH ZKLFKDIHFVWVKH ZDVHUFHPHQW DW LWKHSHUFHQW DJH RI WKH FHPHQW DGGHG WR WKH VWDELOLVHG VRLO DQG WKH FXULQJ FRQGLWLRQ VXFK DV WKH GHJUHH RI WHPSHUDWXUH DQG WKH FXULQJ SHULRG>@7KHGHYHORS PHQWLQWKHVWUHQJWKRI VWDELOLVHGVRLOLV GHSHQDQW PDLO\ORQWKH K\VLFDQDQGLFKHPFDQ OSURSHUWLHVRI WKH WUHDWHGVRLODQGLQWKH\SHRILVWPLQHUDO V7KHH[SHULPHQWDO ZRUNV FRQGXFWHG WR PHDVXUH WKH FRPSUHVVLYH VWUHQJWK RI FHPHQWVWDELOLVHGVRLOLVYHU\HVVHQWLDQWR

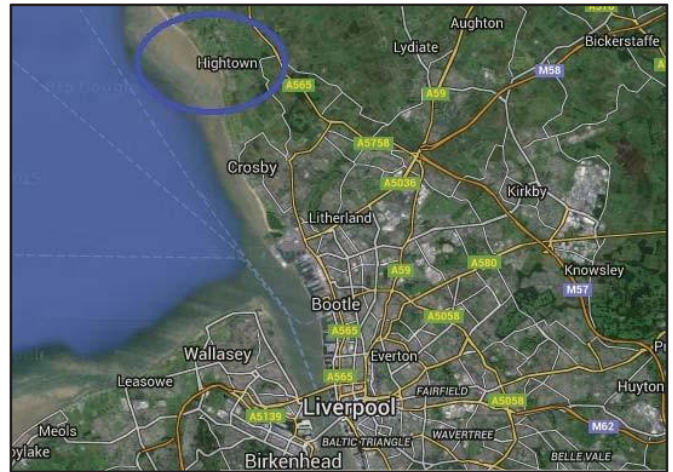
SUHGLFWWKHVLQVLWX  
VRLOVWUHQJWKDIWHUWUHDWLOJZLWKFHPHQW+R  
ZHYHUWKHHIHFWR  
WKHFPHQW\SHRQWKHLPSURYHHPHQWSURFHVVZ  
DVLJQRUHGLQ

7KLV SDSHU IRFXVHV RQ WKH LQYHVWLJDWLRQ  
RI JHRWHFKQLFDO SURSHUWLHV RI  
FHPHQWVWDELQVHG VRIW VRLO ZLWK GLIIHUHQW  
SHUFHQWDJHVDQGRIWKHGUVZHLJKWDQG FXUH  
IRU GLIIHUHQW SHULRGV RI FXULQJ ZLWK VKRUW  
DQG ORQJ  
SHULRGV DQG D\ \$GGLWLRQDOO\ WKLVSDSHU  
SUHVHQWVWKHGHYHORSHPHQWRIDVWDLVWLFDO  
PRGHQVLPXOWL  
UHUHVVLQRPRGHQOLQJWRHYDOXDWHWKHUHOD  
WLRQVQLSEHWZHHQWKH PHDVXUH DQG WKH  
SUHGLFWHG RI 8&6 YDOXH XVLQJ 6366 SURJUDP 7KH  
HIIHFWR RI 23& WUHDWPHQW RQ WKH FRQVWVHQW\  
OLPLVW DQG WKHFRPSDFWLRQSDUDPHWHUVRIWKH  
VWDELQVHGVRLOLQ  
WKLVVWXG\ZHUHDOVRLQYHVWLJDWHG

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*A. Soil Sample*

7KHWUHDWHGVRLOZDVH[SURUWHGIURPWKHEDQN  
RIWKH5LYHU  
\$OWZKLFKLVORFDWHGLQ+LJKWRZQWRWKHQRUWK  
RI/LYHUSRRLOQ WKH 8. 7KLV W\SH RI VRLO FRYHUV  
D YHU\ ODUJH DUHD RI WKH  
+LJKWRZQDVZHOODVWKHFRDWDODUHDRIWKHUL  
YHUZKLFKFDQEH  
XVHG DVIXOOPDWHULDOV LQGLIIHUHQWFRQVWUXF  
WLRQSURMHFWV)LJ  
VKRZVWKHPDSVRI/LYHUSRRODQGWKHFRUHVSRQ  
GLQJVLWHZKHUH  
WKHVRLOVDPSONVXVHGLQWKLVVWXG\ZHUUH[WU  
DFWHG  
7DEOH, LOOXVWUDWHVWKHPDLQSK\VLFDODQGF  
KHPLFDORIWKHVRIW  
VRLOXVHGLQWKLVVWXG\,WFDQEHVHHQWKDWWK  
HVRLOLVFRPSRVHG  
IURPVDQGVLOWDQGFOD\LTXLGOLPLW  
//DQG,3ZHUHPHDVXUHGDQGWKHZHUHIRXQGWREH  
HTXDWR DQG UHVSHFWLYHO\ JURP YDOXH RI  
WKH SDUDPHWHUV LOOXVWUDWHG LQ 7DEOH , DQG  
DFRUGLQJ WR WKH 8QLILHG 6RLO  
&ODVVILFDWLRQ6\ VWHP86&6WKHVRIWVRLOXVHG  
LQWKLVVWXG\  
LVDQLQWHUPHGLDWHSDVWFLW\VLQV\FOD\ZLW  
KVDQG&

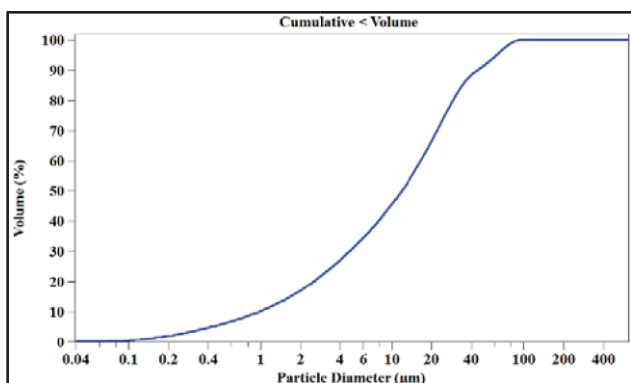


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WKHVRIWVRLOXVHGLQ  
WKLVVWXG\

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3ODVWFLW\,QGH[3,	
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0D[LXP'U\HQVLW'0"JFP	
2SWLXP'PRLVWXUHFRQWHQW20&	
S+	
2UJDQLF0DWWHU&RQWHQW	
8QFRQLQH&RPSUHVVLVH6WUHQJWKIRU	
8QGLVWXUEHG6RLOTXN3D	
JFP JUDPFELFFHQWLPWHUN3D NLORSDFDO	

*B. OPC*

7KH23&XVHGLQWKLVVWXG\ZDVDFRPPHUFDOO\DY  
DLODEOH FHPHQW W\SH &(0,,\$/ 1 DQG LW ZDV  
EURXJKW IURP  
&HPH[4XDOLW\HSDUWPHQW:DUZLFNVKLUH8.)LJVK  
RZV WKH SDUWLFOH VL]H GLVWULEXWLRQ FXUYH  
RI WKH 23& XVHG LQ WKLV VWXG\ ZKLFK ZDV  
REWDLQHG IURP XVLQJ WKH ODVHU SDUWLFOH  
DQDO\VHU7KHPDMRUR[LGHVRIWKHXVHG23&DUHO  
LVWHLQ7DEOH  
„ZKLFKZHUHPHDVXUHGE\FRQGXFWLQJWKH;5D)OR  
UHVHFWHG  
6SHFWURPHWU\WHVW;5)



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QWXVHGLQWKLWVWXG\

)URP)LJLWFDQEHVHHQWKDWKHSUWLF0HVRI  
WKH23&  
XVHGLQWKLWVWXG\DUHUDQJLQJEHWZHHQDQDSS  
UR[LPDWHO\ PLFURPHWUHV ZKLFK ZRXOG  
HQKDQFH WKH SR]]RODQLF

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3HUFHQWDJH				

C. Statistical Program

\$ PXOWL UHJUVVLRQ IXQFWLRQDO RI WKH  
VWDWLWVWLFDO SURJUDP  
6366ZDVXVHGLQWKLWVWXG\7KHUHVXOWVRI8&6W  
HVWZDV XWLOLVHG LQ WKLW SURJUDP WR ILQG D  
UHODWLRQVKLS EHWZHHQ WKH PHDVXUHG DQG  
SUHGLFWHG T<sub>x</sub> RI WKH VRLO WUHDWHG ZLWK  
GLIIHUHQW SHUFHQWDJHV RI 23& DQG H[SRVHG  
WRGLIIHUHQW FXULQJSHULRG  
6366ZDVXVHGWGRGHYHORSQHZVWDLWVWLFDOPR  
GHOWRHYDOXDWH 8&6 GHSHQDQW RQ WZR  
IDFWRUVSHUFHQWDJH RI WKH DGGHG  
FHPHQWDQGWKHOHQJWKRIFXULQJSHULRG

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A. Methodology

SIWHUWKHLGHQWLILFDWLRQRIWKHPDWHULDOV  
XVHGLQWKLWVWXG\  
VRLOVDPHOVZHUHDLUGULHGWKHQSOYHULVHG  
WRVPDOOSDUWLF0HV XVLQJ ZRRGHQ KDPPHU WR  
EH UHGD\ IRU H[SHULPHQWDO ZRUNV  
7KH LQYHVWLJDWHG JHRWHFKQLFDO  
SDUDPHWHUV RI WKH FHPHQW  
VWDELOLVHGVVRLOXVHGLQWKLWVWXG\ZHUFHPSD  
FWLRQSDUDPHWHUV  
7KHVSHFLPHQVRI8&6WHVWZHUHSUHSUHG\SUH  
VVLQJWKH VRLOFHPHQW SDVWH LQ IL[HG YROXPH  
PRXOG XVLQJ PDQXDO  
K\GUDXOLFMDFNWRSURGXFHVSHFLPHQVZLWKVWD

QGDUGGLPHQVLRQ  
PPLQGLDPHWHUDQGPPLQKHLJKWDVVKRZQLQ)LJ  
7KHQ WKH VSHFLPHQV ZHUH ODEHOHG DQG  
ZUDSSHG ZLWK FOLQJ  
IOPWKHQVWRUHGLQKXPLGLW\FDELQHWIRUFUL  
QJXQGHU<sup>R</sup>&  
LQWHPSHUDWXUHDQGXPLGLW\8&6WHVWZDVFRQ  
GXFWHG XVLQJ D WULD[LDO WHVWLQJ PDFKLQH E\  
PDLQWDLQLQJ WKH ODWHUDO SUHVXUH WR EH  
HTXDO WR ]HUR DQG LQ DFFRUGDQFH WR %ULWLVK  
)L3UHSULQJRI8&6WHVWVSHFLPHQV

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A. Results of Experimental Works

7KH HHHFW RI FHPHQW VWDELOLVWLRQ RQ WKH  
FRPSDFWLRQ  
SDUDPHWHUVRIWKHVRLOWUHDWHGLQWKLWVWXG  
(LVVKRZQLQ)LJ  
,WFDQEHVHHQWKDWWKH0"GHFUHDVHGDQGWKH20  
&LQFUHDVHG  
ZLWKWKHLQFUHDVHRIWKHDPRXQWRIWKHDGGHG  
FHPQWVRWKH VRLO 7KLV LV DWULEXWHG WR  
WKH IORFFODWLRQ DQG WKH  
DJORPHUDWLRQRWRNSODFHEHWZHHQFOD\SDUWL  
FOHVLQWKHVRLODQG FHPHQW )LJ VKRZV WKH  
UHODWLRQVKLS EHWZHHQ WKH DGGHG  
SHUFHQWDJHRI23&DQGWKHFHQWVWVWHQF\OLPLWVR  
IWKHVWDELOLVHG  
VRLO,WFDQEHVHHQWKDWWKHYDOXHRI/LQFUHDV  
HGILVWZLWK  
UHVXOWVRI8&6WHVWLQGLFDWHGKDWVRLOVW  
UHQJWKZDVLPSURYHG  
VLJQLILFDQW\HVSFLDOO\IRUWKH23&SHUFHQWD  
JHVEHRQJ  
+RZHYHUQVVLJQLILFDQWVPSURYHHPHQWLQ8&6ZHU  
HREHVUYHG  
IRUFULQJSHULRGORQJHUWKDQGD\VRUDOOSHUFH  
QWDJHVRI  
23&2YHODOOWKH8&6RIWKHVRIVRLOXVHGLQWKL  
VWXG\ZDV LPSURYHG VLJQLILFDQW\ ZLWK WKH  
FHPHQW WUHDWPHQW ZKHQ LW  
LQFUHDVHGIURPN3DIRUXQUHDWHGVVRLOWRN3DZ  
LWKWKH  
XVHRI23&DQGDIVHUGD\VRIFXULQJ  
H[SHULPHQWDO ZRUN 7KLV PRGHQ KDV EHHQ  
GHYHORSHG XVLQJ PXOWLSOH UHJUVVLRQ  
WHFKQLTXH 7KH ODWHU FDQ EH GHILQH DV  
SDFN RI WHFKQLTXHV WKDW KHOSV WR GHYHORS D  
UHODWLRQVKLS EHWZHHQ D GHSHQGHQW  
YDULDEOH ZLWK VHW RI LQGHSHQGHQW  
YDULDEOHV >@ ,Q IDFW PXOWLSOH UHJUVVLRQ  
WHFKQLTXH LV QRW  
DSSOLFDEOHIRUVPDOOVHVVIRIGDWDQGWKHUHP  
RUHWKDRQHNV\ DVVXPSWLRQ PXVW EH FKHFN  
EHIRUH SHUIRULQJ WKLW WHFKQLTXH

ZKHUH>@UHFRPPHQGHGWKHIROORZLQJIRUPXODW  
RFDQXODWH  
WKHPLQLPXPHTXLUHGGDWDVLJH

)LQDOO\WKHFRQWULEXWLRQRHDFKLGHSHQGH  
QWYDULDEOHWRWKH FRQVWUXFWHG PRGHO KDV  
EHHQ LQYHVWLJDWHG E\ FDOFXODWLQJ LWV  
VWDWLWVLFDO VLJQLILFDQW*Sig.* DQG *Beta* YDOXH  
7KH WHUP *Sig.*  
LQGLFDWHVZKHWKHUWKHVWXGLHGLQGHSHQGHQ  
WYDULDEOHFRQWULEXWHV  
WRWKHPRGHORUQRWDQ\LGHSHQGHQWYDULDEO  
HXLW*Sig.* YDOXH  
PRUHWKDQZLOOQRWFRQWULEXWHWRWKHPRGHOE  
XWLWSOD\DQ  
LPSRUWDQWUROHLQWKHSUHGLFWLRQSURFHVZK  
HQLWV*Sig.*YDOXHLV OHVV WKDQ >@ :KLOH *Beta*  
YDOXH LQGLFDWHV WKH UHODWLYH LPSRUWDQFH  
RI HDFK LQGHSHQGHQW YDULDEOH ZLWKLQ WKH  
FRQVWUXFWHG PRGHO WKH KLJKHU WKH *Beta*  
YDOXH WKH KLJKHU  
UHODWLYHLPSRUWDQFH>@*\$*FRUGLQJWRWKHUHV  
XOWVRI7DEOH,, DOO WKH VWXGLHG SDUDPHWHUV  
VLJQLILFDQW\ FRQWULEXWH WR WKH  
FRQVWUXFWHGPRGHODVWKHLU*Sig.*  
YDOXHVDUHOHVVWKDQ\$WKH VDPH WLPH LW  
FDQ EH VHHQ WKDW WKH 23&LQGHSHQGHQW  
YDULDEOHSOD\VKHPRVWLPSRUWDQWUROHLQWK  
HPRGHODVLWV*Beta*  
YDOXH LV DERXW WKUHH WLPHV WKDW RI DJH )LJ  
VKRZV KLJK

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7KLVVWXG\HYDOXDWHVWKHJHRWHFKLFDOSDUD  
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HH[SHULPHQWDOZRUV IRU FRPSDFWLRQ  
SDUDPHWHUV FRQVLVWHQF\ OLPLWV DQG 8&6  
\$GGLWLRQDOO\WKLVSDSHUSURYLGHVVDWDLVW  
LFDOPRGHORSUHGLFW WKH FRPSUHVVLYH  
VWUHQJWK RI WKH VWDELOLVHG VRLO IRU  
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SHUFHQW DJH VRI23&DQGOHQJWKVRIFXULQJSHULR  
GGHSHQGDQWRQ WKH PHDVXUHG GDWD IURP WKH  
H[SHULPHQWV \$FRUGLQJ WR WKH  
ILQGLQJVRWKLWUHVDUFXVWXG\WKHIROORZLQJ  
FRQFOXVLRQVFDQ EHGUDZQ  
x 7KH UHVXOWV RI FRPSDFWLRQ WHVW LQGLFDWHG  
WKDW 0"  
GHFUHDVHGZLWKLQFUHDVHRIWKHGGHSHUF  
HQW DJH RI23&  
2QWKHRWKHUKDQGWKH20&LQFUHDVHGZLWKLW  
HLQFUHDVH

RI23&GXHWRWKHLQFUHDVHRIZDWHUGHPDQGIRUF  
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x&HPHQWVWDELOLVWLRQZDVIRXQGYHU\HIIHFWL  
YHWRLPSURYH  
\$WVHUEHUJOLPLWVRI+LJKWRZQ\VVRIWVRLQ%RW  
KRI//DQG 3/ LQFUHDVHG ZLWK LQFUHDVH RI 23&  
EXW WKH LQFUHDVHG RFFXUUHG LQ // ZHUH  
OHVV WKDQ WKRVH IRU 3/ WKXV WKH YDOXH RI ,3  
GHFUHDVHG ZLWK OHDG WR HQKDQFH WKH VRLO  
VWUHQJWK DJDLQVW WKH VWUHVHV RI  
VZHOOLQJ DQG VKULQNDJH  
2YHUODO,3ZDVGHFUHDVHG IURPWRE\XVLQJ 23&  
x7KHUHVXOWVRI8&6WHVWLQGLFDWHG DQGLQFUHD  
VHLQWKHVRLO  
VWUHQJWKZLWKFRQWLQXRVLQFUHDVHLQ23&D  
VZHOODVLQWKH  
LQFUHDVHRIFXULQJWLPH\$GGLWLRQDOO\WKHUHV  
XOWVRI8&6  
WHVWUHYHDOHGWKDWWKHUHZDVFOHDUGHYHO  
RSPHQWLQWKHVRLO  
VWUHQJWKEH\RQGRI23&+RZHYHUWKHPRVWRIWKH  
GHYHORSHPHQWV JDLQHG LQ VRLO VWUHQJWK  
ZHUH DFKLHYHG  
EHWZHHQDQGGD\VRIFXULQJ  
x:LWK UHVSHFW WKH HDUO\ DJH VWUHQJWK DQ  
GD\ WKH UHVXOWV LQGLFDWHG WKDW  
VLJQLILFDQW LPSURYHPHQW ZDV  
DFKLHYHGIRUVRLOWUHDWHGZLWKDQGDERYHRI2  
3&  
x7KH FRQVWUXFWHG PRGHO VKRZHG D YHU\ JRRG  
UHSURGXFLELOLW\ ZLWK 5 RI ,Q DGGLWLRQ WKH  
FRPSDULVRQ EHWZHHQ WKH SUHGLFWHG DQG WKH  
H[SHULPHQW UHVXOWV LQGLFDWHG WKDW  
WKH FRQVWUXFWHG PRGHO FDQ EH  
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IRU H[SHULPHQWDO ZRUN  
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OLNHWRH[SUHVVWKHLUDFNQRZOHGJPHQWWRWKH  
,UDTLPLQLVW\RI KLJK HGXFDFWLRQ DQG  
VFLHQWLILF UHVHDFK WKH 8QLYHUVLW\ RI  
%DE\ORQ FROOHJH RI HQJLQHHLQJ%DE\ORQ ± ,UDT  
DQG WKH  
8QLYHUVLW\RI:DWLW)DFXOW\RIHQJLQHHLQJ:DWLW  
,UDTIRU WKH ILQDQFLDO VXSSRUW IRU WKLV VWXG\  
\$OVR WKH ILUVW DXWKRU ZROG OLNH WR  
H[SUHVV KLV GHHS JUDWLWXGH WR WKH WKLUG  
DXWKRU  
'U:LOOLDP\$WKHUWRQIRUKLVVXSHUYLVLRQDQGVX  
SSRUWIRUWKLV

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0RGDUUHV\$DQG1RVRXG\<0&OD\6WDELOLVDWLRQ8VLQJ&RDO

:DVWHDQG/LPH±7HFKQLFDODQG(QYLURQPHQWDO,PSDFW*Applied Clay Science*SS

-DIHU+0\$WKHUWRQ:5XGGRFN) /RIILO(\$VVHVVLQJ  
WKH3RWHQWLDORID:DVWH0DWHULDOIRU&HPHQW5HSODFHP  
HQWDQGWKH

(IHFWRI,WV)LQHQQHVLO6RIW6RLO6WDELOLVDWLRQ,QWHUQ  
DWLRQDO-RXUQDORI (QYLURQPHQWDO &KHPLFDO

(FRORJLFDQ \*HRORJLFDQ DQG \*HRSK\VLFDQ (QJLQHHULQJ

-DXEHUWKLH 5 5HQGHOO ) 5DQJHDUG ' DQG 0ROH] /

6WDELOLVDWLRQRIHVWVDULQHVLOWZLWKOLPHDQGRUFHP

HQW\$SSOLHG&OD\ 6FLHQFH

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2QDO 2 /LPH 6WDELOLJDWLRQ RI 6RLOV 8QGHUO\LQJ D 6DOW  
(YDSRUDWLRQ 3RQG \$ /DERUDWRU\ 6WXG\ 0DULQH

\*HRUHVRXUFHV \*HRWHFKQRORJ\

5LRV6&ULVWHOR19LDQDGD)RQVHFDSDQG)HUUHLUD&

6WUXFWXUDO 3HUIRUPDQFH RI \$ONDOL\$FWLYDWHG 6RLO

\$VK YHUVXV 6RLO

&HPHQW-RXUQDORI0DWHULDOVLQ&LYLO(QJLQHHULQJ

0DNXVD\*36RLO6WDELOLJDWLRQ0HWKRGVDQGD0DWHULDOVL

Q

(QJLQHHULQJ3UDFWLFH/XOHn6ZHGQ/XOHn8QLYHUVLW\RI7H

FKQRORJ\

(VNLVDU7,QIOXHGFHRI&HPHQW7UHDWPHQWRQ8QFRQILQH

&RPSUHVVLYH6WUHQJWKDQ&RPSUHVVLELOLW\RI/HDQ&OD

\ZLWK0HGLXP

3ODVWLFLW\SUDELQ-RXUQDOIRU6FLHQFHDQ(QJLQHHULQJ

/RQGRQ8.%ULWLK6WDQGDUGLQVWLWXWH

%RVWRQ\$OO\QDQ%DQFRQ