

## Advancements in Quantum Harmonics

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**Abstract** 7KLVS DSHUSUH VHQWVWKHHIIFWRIFRQFUHW  
HFRPSUHV VLYH  
VWUHQJWKDQGUHFWDQJXODULWUDWLRQVWUHQJ  
WKDQGGXFWLWLRQRIQRUPDO DQG KLJK VWUHQJWK  
UHLQIRUFHG FRQFUHWH FROXPQV FRQILQH ZLWK  
WUDQVYHUVH VWHHO XQGHU D[LDO FRPSUHV VLYH  
ORDGLQJ ILQWHHQ Qrupdo  
VWUHQJWKFRQFUHWHUHFWDQJXODUFROXPQVZLWKG  
LIIHQWYDULDEOHVWHVWHG  
LQWKLWUHVHDFKZHUHVHGWRVWXG\WKHHIIFWRI  
FRQFUHWHFRPSUHV VLYH  
VWUHQJWKDQGUHFWDQJXODULWUDWLRQVWUHQJ  
WKDQGGXFWLWLRQRIQRUPQV  
7KHS DSHUDOVRSUHVHQWVDQRQOLQHDUILQLWHHOH  
PHQWDO\VLVIRUWKHVH  
VSHFLPHQV DQGDQRWKHUWZHQW\KLJKVWUHQJWKFR  
QFUHWHV TXDUHFROXPQV  
WHVWHGE\RWKHUHVHDFKHUVXVLQJ\$16<6ILQLWH  
HOHPHQWVRIWZDUH 7KH UHVXOWV LQGLFDWH WKDW  
WKH D[LDO IRUFH ± D[LDO VWUDLQ UHODWLRQV KLS  
REWDLQHG IURP WKH DQDO\WLFDO PRGHO XVLQJ \$16<6  
DUH LQ JRRG  
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DULVRQV KRZVWGDW WKH  
\$16<6LVFDSDEOHRIPRGHOLQJDQGSUHGLFWLQJWKHDF  
WXDOQRQOLQHDU EHKDYLRU RI FRQILQH Qrupdo  
DQG KLJKVWUHQJWK FRQFUHWH FROXPQV XQGHU  
FRQFHQWULF ORDGLQJ 7KH PD[LXP DSSOLHG ORDG  
DQG WKH PD[LXP VWUDLQ KDYH DOVR EHHQ  
FRQILUPHG WR EH VDWLVIDFWRU\ 'HSHQGLQJ RQ WKLV  
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VWXG\ ZDV FRQGXFWHG E\ \$16<6 WR FODUL\ DQG  
HYDOXDWH WKH HIIHFWRU RI HDFK YDULDEOH RQ  
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**Keywords** \$16<6 FRQFUHWH FRPSUHV VLYH VWUHQJWK  
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GXFWLWLRQ\UHFWDQJXODULWUDWLRVWUHQJWK

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XPQJHRPHWU\ DQG  
WLHFRQILJXUDWLRQV DUHDVVKRZQLQJLQDQGWKH  
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### A. Evaluation of Tested Columns

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### B. Evaluation of Column Strength

DQGXQFRQILQHGFROXPQVUHLQIRUFHWHV VZDVFRPSXWHGE\  
7KH  
FRUHFDSDFLW\ZDVHVWDEOLVKHGE\XEWUDFWLQJ  
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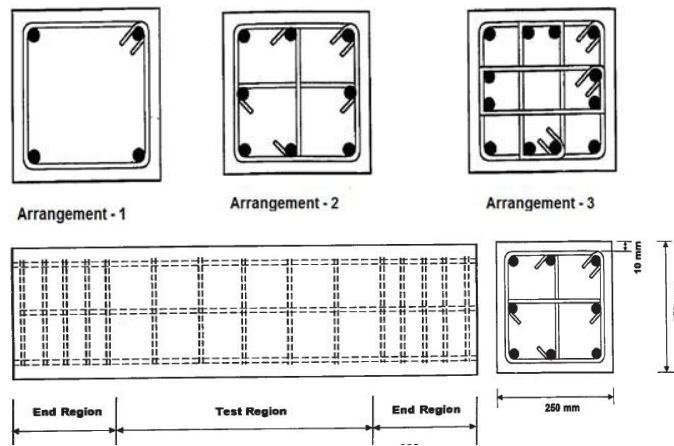






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RGHOWKH  
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LWKWKUHHGHJUHHV  
RIIUHHGRPDWHDFKQRGH±WUDQVODWLRQVLQWKH  
QRGDO[DQG] GLUHFWLRQV 7KH HOHPHQW LV  
FDSDEOH RI SUHGLFWLQJ SODVWLF GHIRUPDWLRQ  
FUDFNLQJ DQG FUXVKLQJ LQ WKUHH RUWKRJRQDO  
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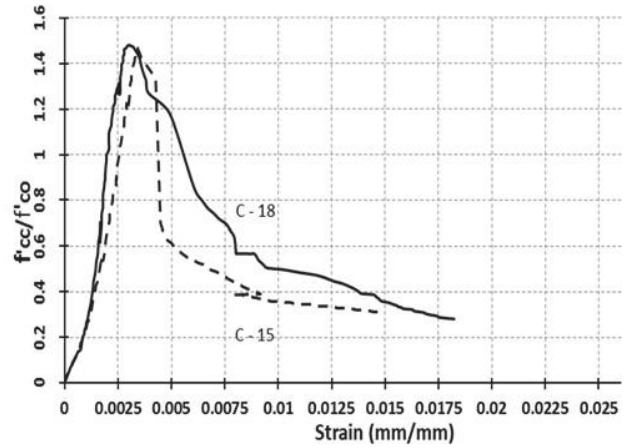
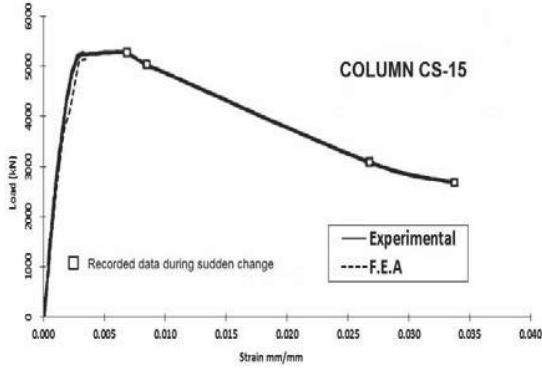


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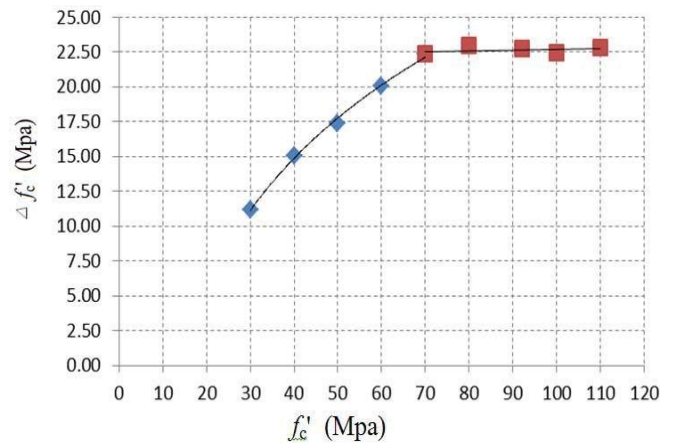
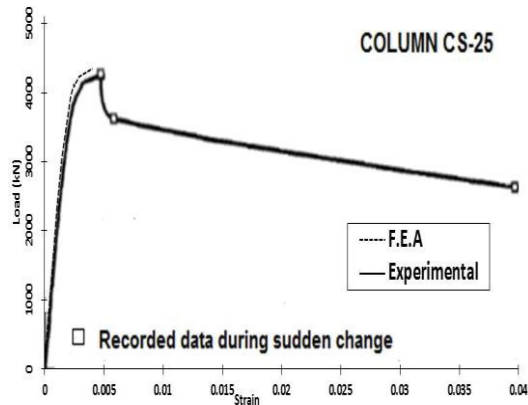
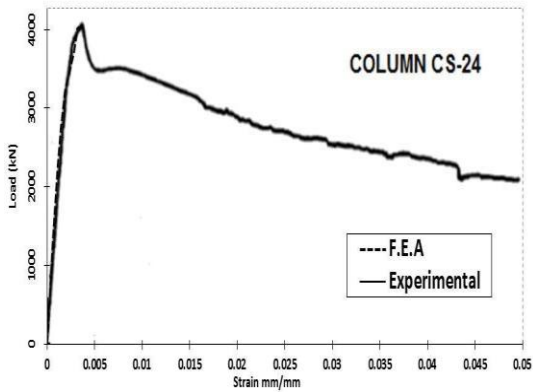
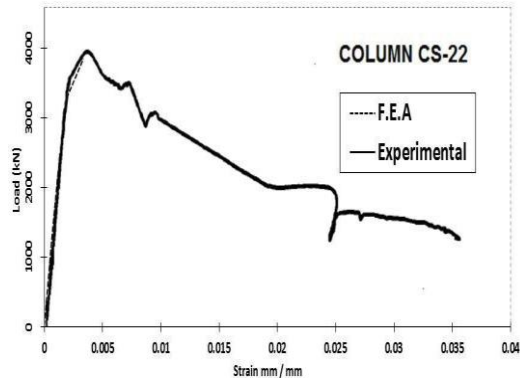
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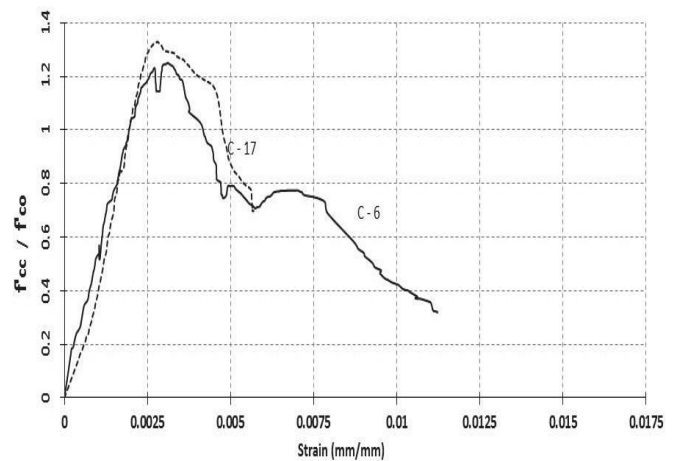
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B. Aspect Ratio

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VTXDUH FROXPQ VKRZHG EHWWHU HQKDFHHPHQW PRUH WKDQ  
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